CR-82159

N67-18776

TM 312-620
Addendum No. 1
2/17/66

code-1 page-30 cat-30

I. Introduction

Section IV of TM 312-620 promised that accuracy results for perturbed Lunar Orbiter trajectories would be given in a follow-on TM. This memo gives those results as well as a little more material on the unperturbed Kepler orbits. Before going into these matters we should recall the principal points of reference 1.

II. Recapitulation of TM 312-620

Reference 1 gave results of an accuracy study of SPACE for Lunar Orbiter trajectories. The trajectories used no perturbations and the injection conditions were the osculating values of the S110 mission as it entered the final (photographic) orbit. Various combinations of integration step size and number of orbits of flight were tried and the results were compared to double precision evaluations of the two-body orbit equations. The main findings were that 1) the positional errors took place along the flight path direction (or, more accurately, along the orbit itself), 2) the velocity errors took place along the radial direction from the central body, 3) the direction of this error relative to the correct position and velocity depended upon the step size, larger step sizes producing leading-type errors and smaller step sizes lagging type, 4) practically all of the error manifested itself in an error in time of pericenter passage, T_p.

The rest of this TM will be given to 1) some further results on the unperturbed or two-body orbits, 2) the accuracy results for trajectories which include the effects of perturbations, 3) the effect of the variational equations upon running time and 4) some remarks.

III. A Few Further Results on Two-Body Orbits

In Figs. 47-55 of reference 1 some plots of the error in the classical elements a, e, i, Ω, ω, T_P vs. time in orbit were shown. Those plots indicated several things. First, the error in the elements a and e was small but increased linearly with time. Second, the error in the elements i and Ω was very small and though it seemed to increase with time the problem was confused by the fact that the results for $\Delta t = 160$ seconds were actually better than the results for $\Delta t = 20$ and 60 seconds. Third, the error in ω seemed to depend rather strongly on step size with the errors in ω for $\Delta t = 160$ sec. being about 1000 times greater (though they were still quite small— about $.04^\circ$ after 80 orbits) than those for $\Delta t = 20$ and 60 seconds. Fourth, the errors in T_P also depended strongly upon step size, being about 1000 times greater for $\Delta t = 160$ sec. than for $\Delta t = 20$ and 60 sec. but they were not small. They also had a very smooth shape very much like a parabola.

To test these observations it was decided to run a few more trajectories which would run a good deal longer and use step sizes that were a good deal larger. For this purpose 1120 hours of orbiting, which is about 320 orbits, and step sizes of 200, 300, 400 and 500 seconds were chosen. The 400 and 500 second trajectories impacted the moon at about $2^d\ 10^h$ and 13^h respectively after injection. Though SPACE could have been altered to by-pass the impact test this was not done and these trajectories were ignored.

Figure 1 shows the error in a and e for the $\Delta t = 200$ sec. trajectory vs. time in orbit. We see that the error plots extremely close to a straight line for both a and e. Fig. 2 shows the same plots for the $\Delta t = 300$ sec. trajectory. Here we see some curvature showing up and the linearity has begun to break down. These errors are rather like periodic variations of an extremely long period which over fairly wide intervals of time act like secular changes.

Plots for the error in i and Ω were not made for the 200 and 300 sec. trajectories since the error was so small it hardly seemed worth plotting. A careful scanning of the results of the double precision calculations showed that Δi, the error in inclination, increased steadily reaching a maximum value of $.000056699364^{\circ}$ for the 200 sec. trajectory and $.000066802312^{\circ}$ for the 300 sec. trajectory. Comparison of these values with the plots of Figs. 50-52 of reference 1 shows that the errors in i for these very large step sizes are of the same order of magnitude as those of the more commonly used step sizes.

The error in Ω is much the same. The computer program results showed that $\Delta\Omega$ increased to a largest value of $.000034259560^{\circ}$ and $.000291886644^{\circ}$ for the 200 sec. and 300 sec. trajectories respectively. The error for the 200 sec. step size is of the same order of magnitude as those of the more commonly used step sizes and that of the 300 sec. step size is just one order of magnitude greater.

In Fig. 3 we see the plots of the error in ω and T_P for the 200 sec. trajectory. The plot of $\Delta\omega$ is fairly straight though it is slightly concave upward. I have no explanation for the occasional "wiggling." (error?) $\Delta\omega$ for the 300 sec. trajectory, Fig. 4, shows a very definite bending. The graphs of the error in T_P show the same sort of shape found in Figs. 53-55 of reference 1. We see that the error in T_P attains values which are a good deal greater than the period of the reference orbit which is about 12,500 seconds. The reason for this is that the error in the integrated solution becomes so great that the integrated solution actually laps the correct solution. In the case of the 300 sec. step size the integrated solution actually lapped the correct solution some 29 times.

On page 21 of reference 1, third paragraph, the statement is made that the error in T_P is so large that it accounts for virtually all the error. I wished to get an accurate quantitative idea as to how much of the error in SPACE could be assigned to the error in time of pericenter passage and so a computer program was written. This program calculates the contribution to the error in the three coordinates of position caused by errors in each of the six elements. For the elements a, e, i, Ω, ω a first order approximation only was used, that is the first partial derivative of the position coordinate with respect to the element was multiplied by the error in the element to obtain the error in the coordinate. For the element T_P the first order approximation was done as well as an "exact" calculation which used the orbit equations themselves rather than their derivatives. This was necessary in the program in order to handle the relatively large errors in T_P. The results of this program for a few of the trajectory points processed are given in Table 1.

Table 1 shows the percentage fraction of the error in the position coordinates attributable to errors in the individual elements. The first line of each of 1-A, 1-B, 1-C, 1-D shows the table label and the step size used in the integration. The second line in each shows the time in hours after injection at which the calculations were made. Two times were used for all four step sizes: t = 80 and 224 hours. The third line shows the error in the Cartesian components of position. The units here are kilometers and the coordinate system used is that of reference 1. The next seven lines show the percentage fraction of the error in the position coordinates. Thus in table 1-B (Δt = 90 seconds) we see that 1.44% of the .73115 km error in Z at t = 224 hours can be attributed to the error in ω which at that time was about .00108°. The errors in the elements themselves are not shown in the table in order to save space. In transcribing the data from the computer output to the memo the data was rounded to two decimal places for brevity. The last two lines show the totals of the percentage fractions of the error (which ideally should be 100%). There are two totals, labelled S_1 and S_2, because the errors due to errors in T_P were calculated in two ways. The error calculated using a first order approximation is labelled T_{P_1} and that using direct calculations is labelled T_{P_2}. Notice that many of the T_{P_2} figures are terribly inaccurate. This was caused by the inability of the single precision 1620 program to cope with differences in large numbers which were very nearly equal. The adequacy of the first order analysis for most of the data sets is indicated by the sums S_1 being very nearly 100%. Only in Table 1-C under t = 224 hours and in Table 1-D are the S_2 sums more accurate than the S_1. Notice the consistently high percentage fraction figures for T_P— roughly 95% overall.

It should be pointed out that the error in T_P can itself undoubtedly be ascribed to the small error in a. This point has not been pursued so no firm statements can yet be made, but it must be more than coincidence that we see an upward bending of the Δ a curve in Fig. 2 and at the same time see a decreased curvature of the ΔT_P curve in Fig. 4 . Sam Pines pointed out in conversations on 2/10 that this does in fact happen. It seems plausible enough.

IV. Accuracy Study Results for Perturbed Trajectories

Reference 1 covered the accuracy findings for Lunar Orbiter type trajectories which were not subject to perturbations. Since analytical solutions were available for this case many precise measurements of accuracy were possible. When we allow perturbations to enter the problem we have no analytical solutions to resort to and computer programs which produce accurate approximate solutions are hard to come by. One way to partially avoid this problem when working experimentally was mentioned in section IV of reference 1. The method explained in terms of Lunar Orbiter trajectories goes like this: From SPACE we get a set of trajectories which use no perturbations. At some particular epoch after injection we examine the set of solutions consisting of the correct solution gotten analytically and the several integrated solutions. In X-Y-Z space these solutions form some sort of geometric pattern. In our case when the step size is less than 150 sec. or so the solutions form a straight line in space which is colinear with the tangent to the Kepler ellipse at the reference point. The next step is to get from SPACE a set of trajectories which do include perturbations. We consider this set of solutions at the same epoch after injection as was used for the unperturbed set. In X-Y-Z space this set forms some sort of geometric pattern. Now if the two geometric patterns look very much like one another the proposition becomes very credible that the program is just as accurate when computing perturbed trajectories as it is when computing unperturbed trajectories. Let us look at these patterns for the Lunar Orbiter trajectories.

In Fig. 5 we see the orthographic views or projections upon the coordinate planes of the straight line formed by the set of integrated solutions plus the correct solution at t = 80 hours. It should be remembered that the solutions really fall on an arc of the reference ellipse (or very close to it) but the scale is such that to the accuracy of a careful plot this arc appears as a straight line. Small coordinate axes appear next to each view for identification. The X-Y projection is a "top" view, the Y-Z a "front" view and the Z-X a "right side" view. In the drawing we see the points which represent the solutions for Δt = 20, 30, 40, 60, 70, 80, 85, 90 seconds as well as the correct solutions. The selenocentric coordinates of the correct solution are given, and the scale appears in the title block. Recall that the step sizes 20, 30, 40, 50, 60, 70 resulted in lagging-type errors and the step sizes 80, 85, 90 and above result in leading-type errors. Thus the 90 sec. end of the line segment points along the direction of motion. We see that Δt = 40 sec. produces the best results.

Figs. 6 and 7 give the corresponding views for t = 176 and 272 hours respectively. Notice that the 70 sec. solution has moved from outside the 20 sec. solution in Fig. 5 to inside in Figs. 6 and 7 and that the relative positions on the straight line are the same in Figs. 6 and 7.

In Figs. 8, 9 and 10 we see the projections of the geometric pattern formed by the solutions when the perturbing effects of C_{20}, C_{22}, the Earth and the Sun are included. The constants used were

$$C_{20} = -.20711 \text{ E-5}$$

$$C_{22} = .20716 \text{ E-4}$$

$$\mu_{\oplus} = .39860063 \text{ E+6}$$

$$\mu_{\odot} = .13271411 \text{ E+12}$$

We see that this too forms a straight line. There is some difference in the positioning of the points on the line; namely, 70 and 30 lie closer to 90 but this small difference seems rather unimportant. The different orientation of the line segment is due to the perturbations in Ω and ω. We see from Figs. 7 and 10 that the similarity between the "perturbed" and "two-body" "error lines" is very strong at t = 272 hours.

Fig. 14 is useful in getting an overall picture of this "error line." In the top plots we see the variation in "percentage distance" of the individual solutions vs. time in orbit. For example we see that the 80 sec solution maintains an almost constant position of 32.5% of the distance from the 20 sec. solution to the 90 sec. solution. Also the correct two-body solution denoted "0" is the plots maintains a nearly constant 23% position between the 20 and 90 sec. solutions. Notice how the noisy character settles out.

In the lower half of Fig. 14 we see the corresponding plots for the perturbed trajectories which use the perturbations mentioned above. One can almost "see" where the correct solution of the perturbed trajectory would lie.

Figs. 11, 12 and 13 show the three views of the error line for the times of 80, 176 and 272 hours after injection when the perturbing effects of C_{20}, C_{30}, C_{40}, C_{22}, the Earth and the Sun are included. The values of C_{30} and C_{40} used were

$$C_{30} = -.863 \text{ E-4}$$

$$C_{40} = .2628 \text{ E-5}$$

and the other quantities had the same values as before. Again we see that the solutions possess the quality of forming a straight line. The positioning of the points plotted vs. time in orbit in Fig. 15 shows greater differences here than in Fig. 14, and one doesn't feel quite as confident about "seeing" the solution in this case, but close examination does reveal many similarities.

Fig. 16 shows a plot of the length of the line segment connecting the 20 and 90 sec. solutions for the unperturbed trajectories. At alternate four hour intervals we see plotted on top of this curve the corresponding quantity for the two different sets of perturbed trajectories. From this plot and from the other plots we see that, whatever the correct solution might be, the collection of solutions forms a straight line and that straight line has the same length (approximately) for any reasonable perturbations, and that (here comes a small speculation) the correct solution lies somewhere on that straight line.

V. Some Further Running Time Results

During the final stages of this study 17 additional trajectories were run whose running times were recorded. These were different from the trajectories reported in reference 1 in that they used perturbations. From Figs. 68 and 69 of reference 1 we see that the speed of the program for nearly all trajectories using more than about 1500 iterations is about .085 sec/iteration. Including the extra perturbations scarcely altered the speed which it appears from the records is about .089 sec/iteration.

There were six runs whose speed was nearly twice as great having an average speed of .0508 sec/iteration. Fig. 69 of reference 1 also shows some trajectories whose running speed was about .050 sec/iteration. Apparently one is lucky enough now and then to submit trajectory runs when the machine is operating in a different mode or when no one else is using it or some such thing.

Since the running time of the program is quite predictable it seemed that one trajectory run would be enough to determine the effect of integrating the variational equations. The result was a speed of .1348 sec/iteration. This compares to .089 sec/iteration without variational equations. Thus including variational equations causes the program to run 50% longer which is very close to what Ray Harris found on an Earth-Mars trajectory.

VI. Main Conclusions and Remarks

After studying the graphs of reference 1 and this addendum it seems safe to conclude that we can expect the same order of accuracy in the simulation of perturbed lunar orbits as in unperturbed lunar orbits. This study does not allow us to say definitely what the exact magnitude and direction of the error in integrating perturbed orbits is at any given time, but it appears that we could, if we chose, find the straight line in space on which the "true" solution lies by performing the integration with two step sizes. Though it wasn't discussed the same kind of thing could be done with the velocity vector. This approach is of a more academic interest than practical but it does improve one's understanding of the behavior of the program.

Another main conclusion to be drawn is that the integrating error, as it shows up in the conic elements, is such that virtually all of the error appears in T_P and that the error in T_P vs. time in orbit could be well approximated by some sort of second degree curve, probably a parabola. This leads to a thought which could be of considerable practical importance. That is this: The constants of a second degree function which would describe the error in T_P could be included in the ODP as solve-for variables. This would be a useful thing to do in handling data arcs of more than say 40 orbits, and it appears that data arcs of several hundred orbits could be easily handled using large integration step sizes (Δt = 150 to 200 seconds) especially if functions (probably linear functions) were included to handle the changes in a and e, too. Doing this could be a first important step toward including in the model the effects upon the trajectory of the <u>integrator itself</u> in addition to the higher order lunar oblateness terms, the perturbing effect of distant planets, gas leaks, etc., etc., etc.

Pines mentioned on 2/10 that he has a way of analytically accounting for integrating errors and has included them in a trajectory program with much success. His method, being analytic rather than empirical, does not require actual observations and an ODP but can be used in a trajectory program alone. There is apparently a goodly amount of theoretical knowledge of these effects well known to specialists in numerical analysis, and the time perhaps has come for incorporating this knowledge in work-a-day trajectory programs.

JG:mjf

Table 1-A — Δt = 40 seconds

	t = 80 hours			t = 224 hours		
	ΔX = -.0112	ΔY = .0062	ΔZ = .0063	ΔX = .12673	ΔY = -.0447	ΔZ = -.0543
a	2.71	1.18	1.73	.20	-.28	-.05
e	6.20	-4.54	-.67	-1.97	2.05	.16
i	.16	.95	-1.44	.04	.34	-.44
Ω	-1.00	-1.94	0	-.28	-.28	0
ω	4.03	5.62	5.03	.83	.42	.62
T_{P_1}	87.44	99.40	94.77	101.18	97.67	99.70
T_{P_2}	6647.28	7767.88	.73	-243.45	-220.14	-126.31
S_1	99.56	100.67	99.41	100.00	99.92	99.99
S_2	6659.40	7769.15	7343.94	-244.64	-197.61	-126.02

Table 1-B — Δt = 90 seconds

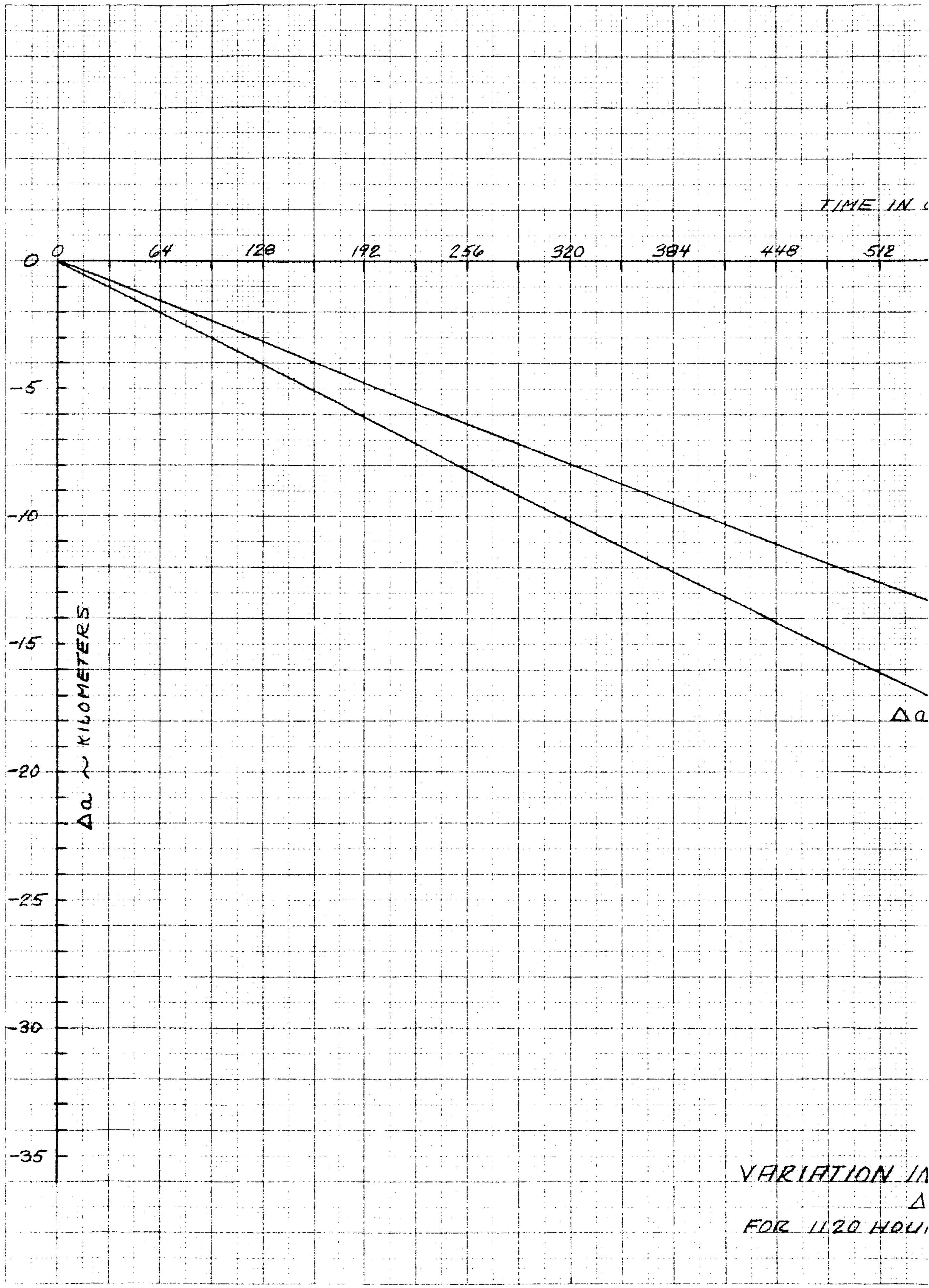
	t = 80 hours			t = 224 hours		
	ΔX = .1092	ΔY = -.0666	ΔZ = -.0652	ΔX = -.173584	ΔY = .5864	ΔZ = .73115
a	6.59	2.60	3.95	.22	-.31	-.06
e	1.44	-.96	-.15	-.32	.35	.03
i	-.02	-.09	.14	0	-.03	.03
Ω	.07	.13	0	.01	.01	0
ω	14.57	18.42	17.12	1.91	1.01	1.44
T_{P_1}	77.38	79.83	79.00	98.32	99.09	98.61
T_{P_2}	692.02	704.82	700.48	96.09	96.73	96.31
S_1	100.04	99.94	100.06	100.14	100.12	100.05
S_2	714.69	724.93	721.54	97.91	97.76	97.76

Table 1-C

	$\Delta t = 140$ seconds					
	$t = 80$ hours			$t = 224$ hours		
	$\Delta X = 9.1422$	$\Delta Y = -5.3815$	$\Delta Z = -5.3324$	$\Delta X = -151.95897$	$\Delta Y = 46.2883$	$\Delta Z = 60.66068$
a	6.04	2.47	3.71	.22	-.35	-.06
e	1.73	-1.19	-.18	-.37	.45	.03
i	0	0	0	0	0	0
Ω	0	0	0	0	0	0
ω	2.92	3.83	3.52	.36	.21	.28
T_{P_1}	89.08	95.22	93.10	98.74	110.37	104.49
T_{P_2}	88.78	94.41	92.46	99.78	99.77	99.77
S_1	99.78	100.34	100.14	98.94	110.68	104.75
S_2	99.48	99.52	99.51	99.98	100.08	100.03

Table 1-D

	$\Delta t = 200$ seconds					
	$t = 80$ hours			$t = 224$ hours		
	$\Delta X = 161.7742$	$\Delta Y = -85.907$	$\Delta Z = -88.3198$	$\Delta X = -2471.7551$	$\Delta Y = -288.687$	$\Delta Z = 311.5742$
a	5.95	2.69	3.90	.23	.96	-.21
e	1.66	-1.26	-.18	-.39	-1.21	.11
i	0	0	0	0	0	0
Ω	0	0	0	0	0	0
ω	-.46	-.67	-.59	-.06	.09	.15
T_{P_1}	89.11	105.59	99.50	105.75	-308.28	354.39
T_{P_2}	92.37	99.66	96.96	100.28	100.49	100.15
S_1	96.25	106.35	102.62	105.53	-308.44	354.14
S_2	99.52	100.42	100.09	100.06	100.33	99.90



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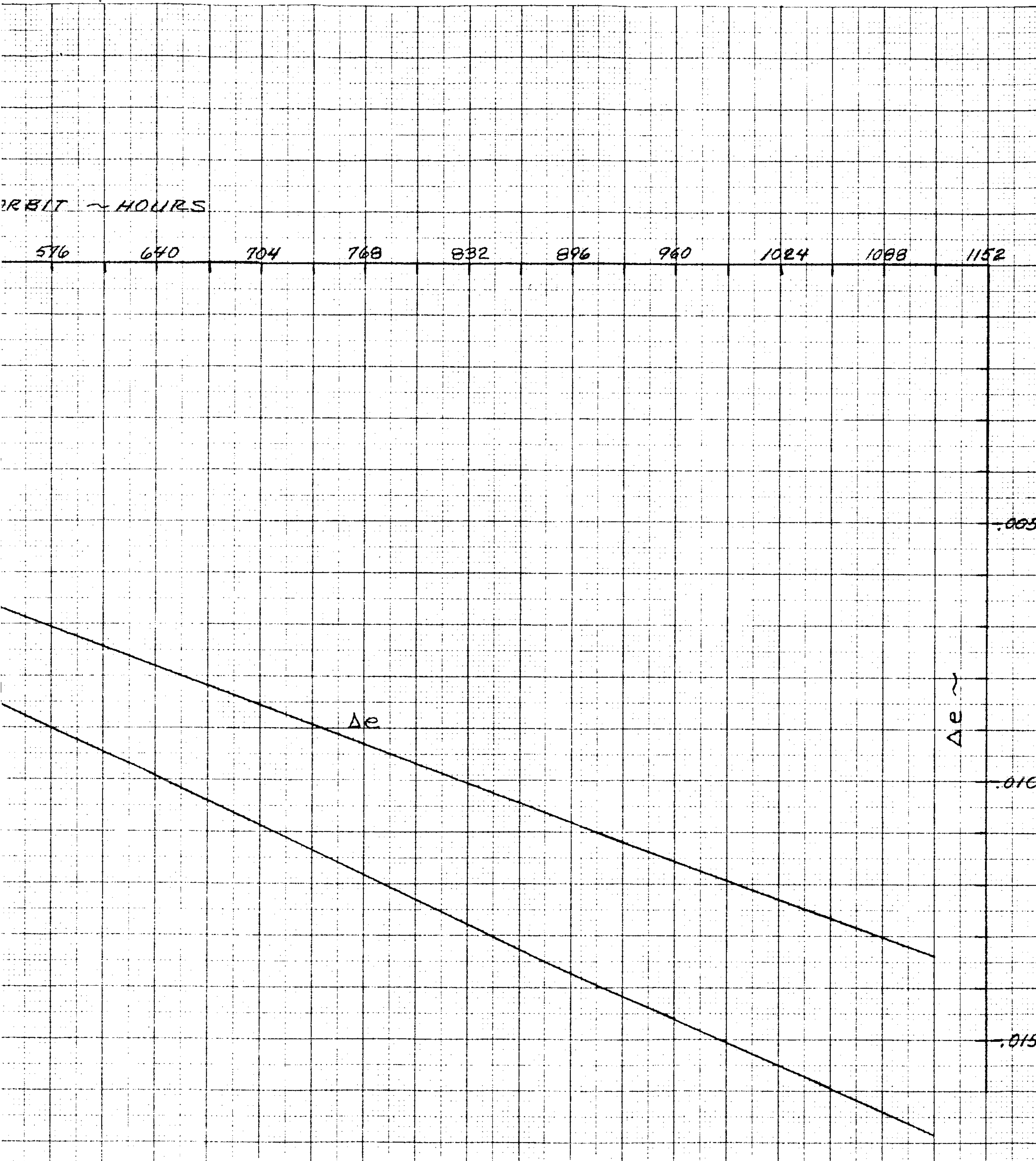

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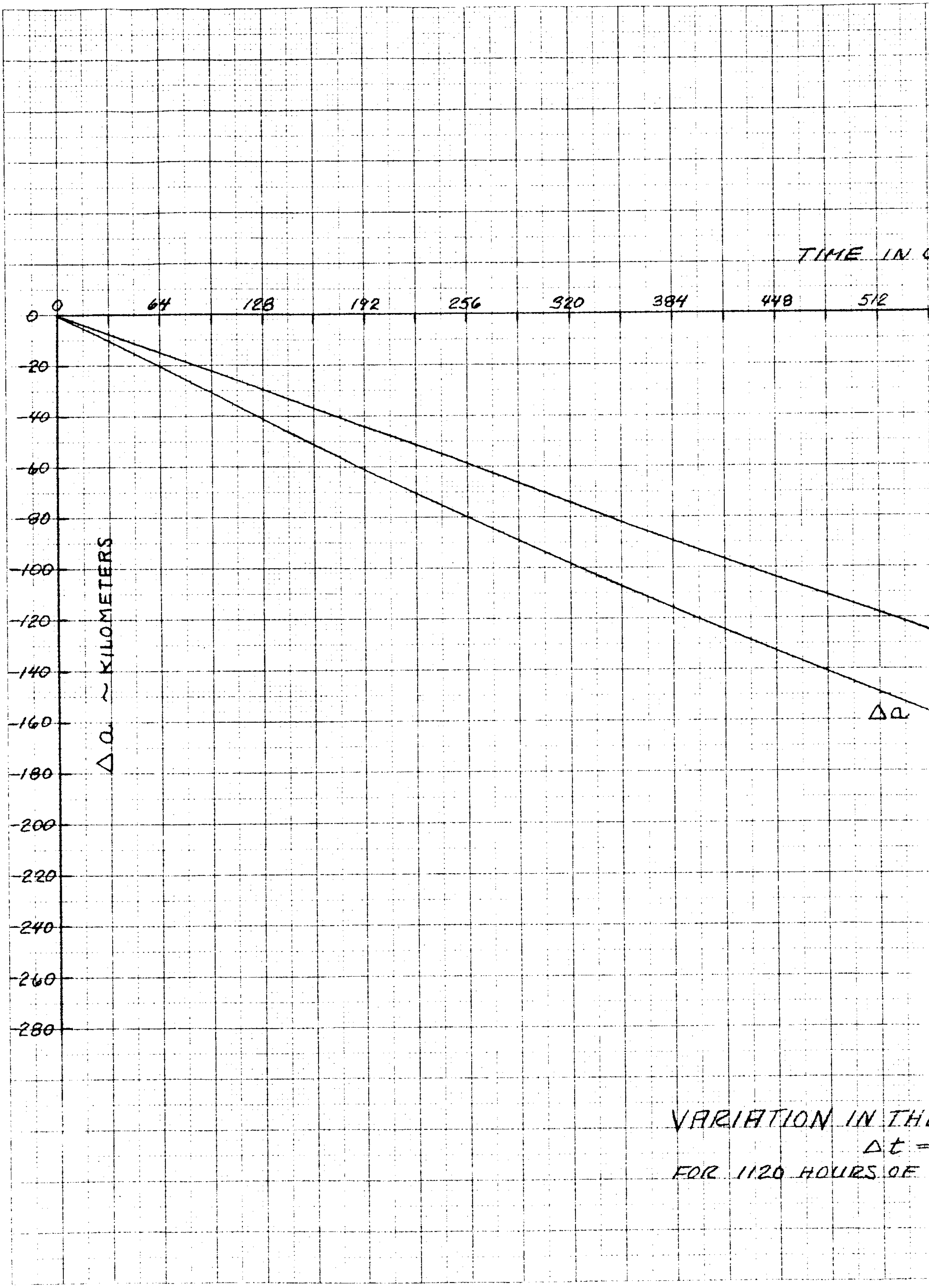
Fig. 1-2



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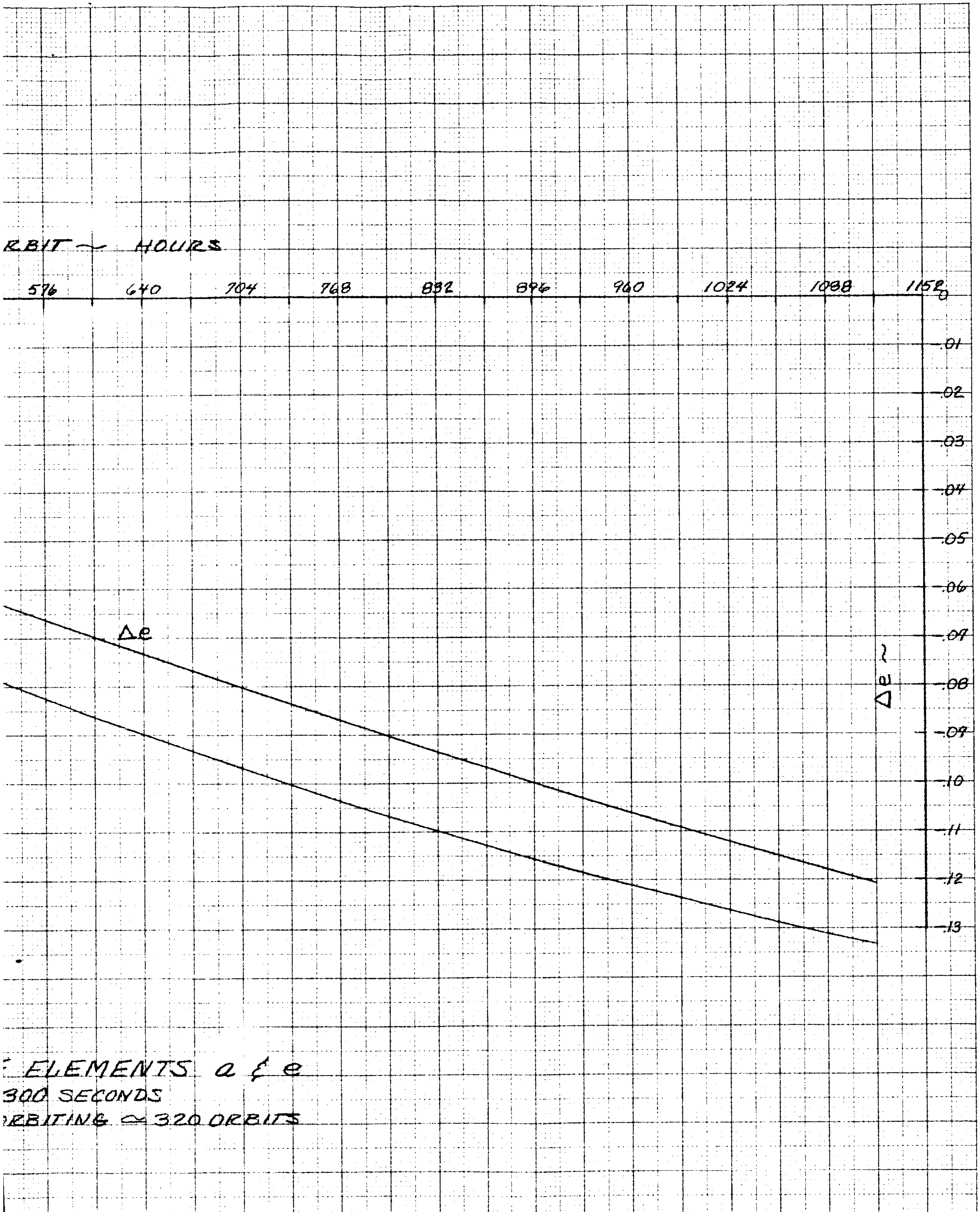

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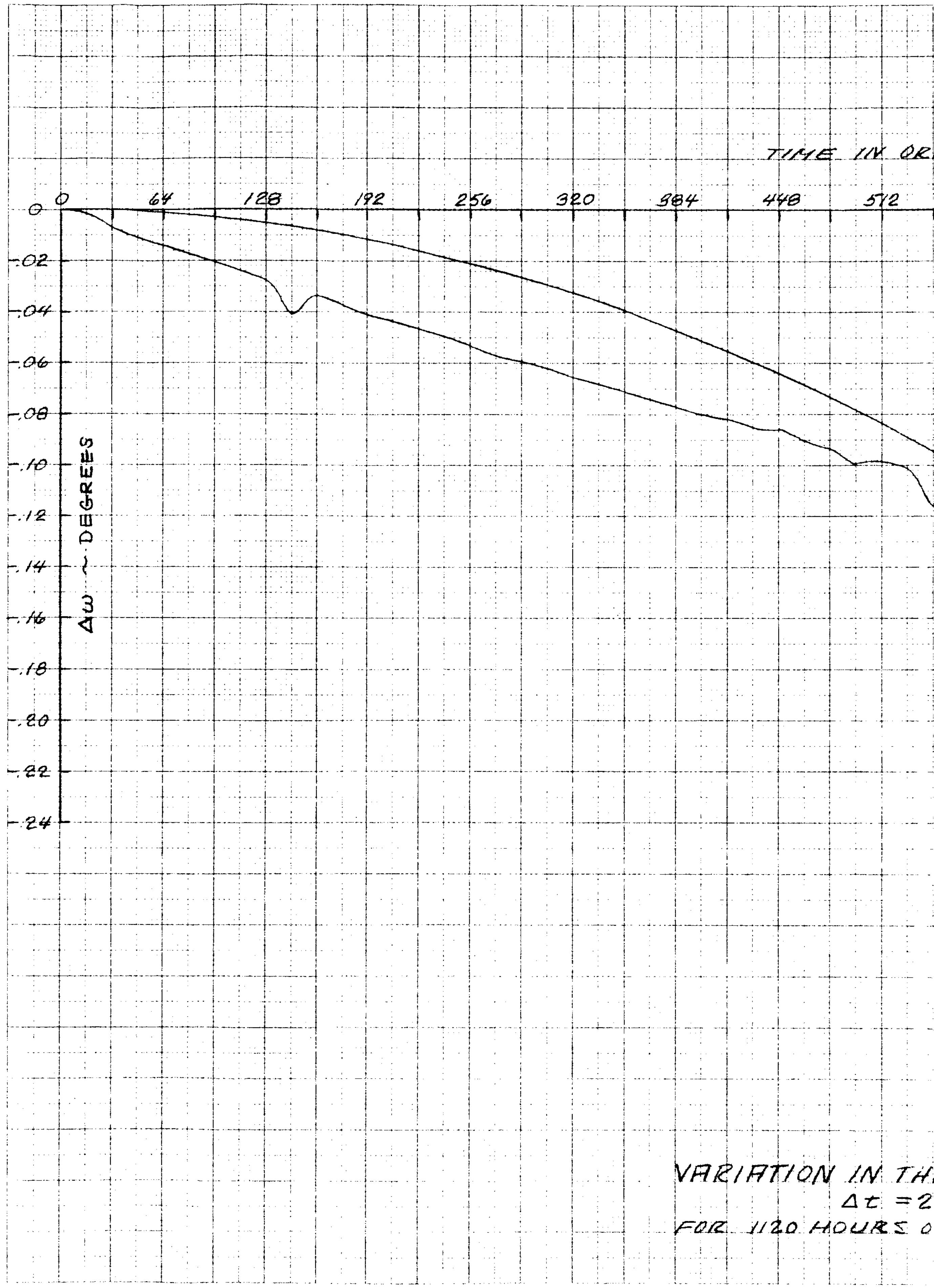
Fig. 2-2



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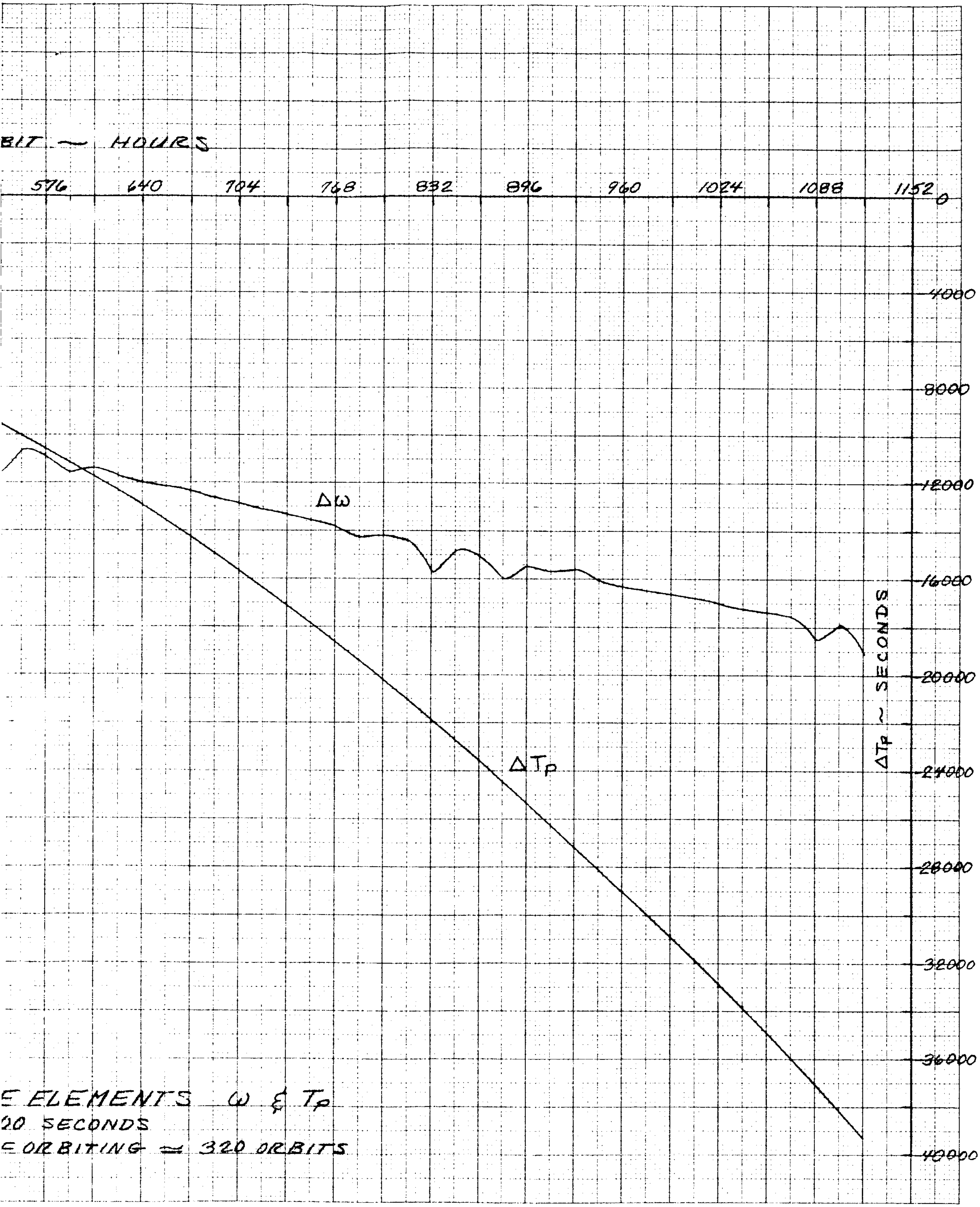
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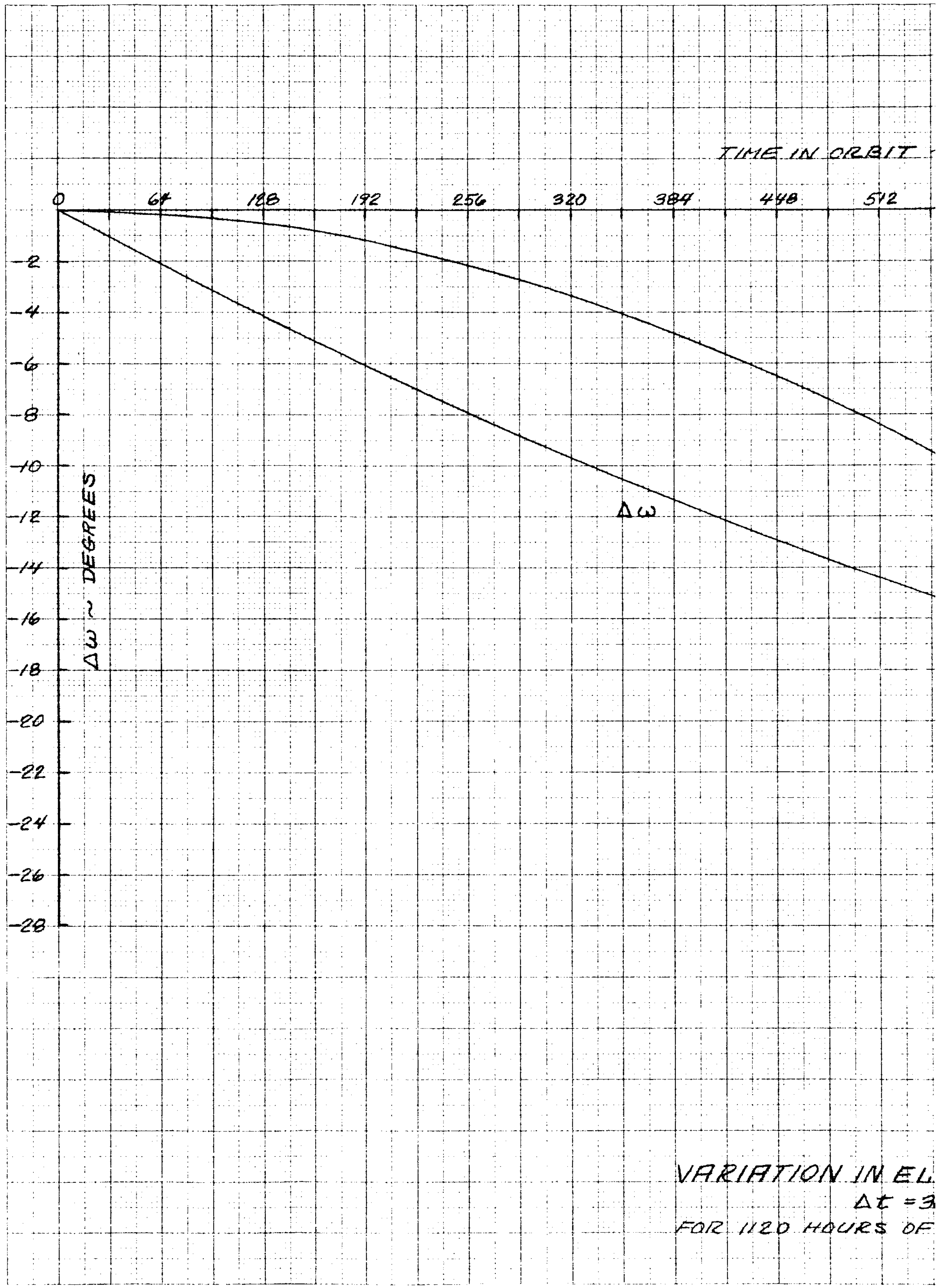
Fig. 3-2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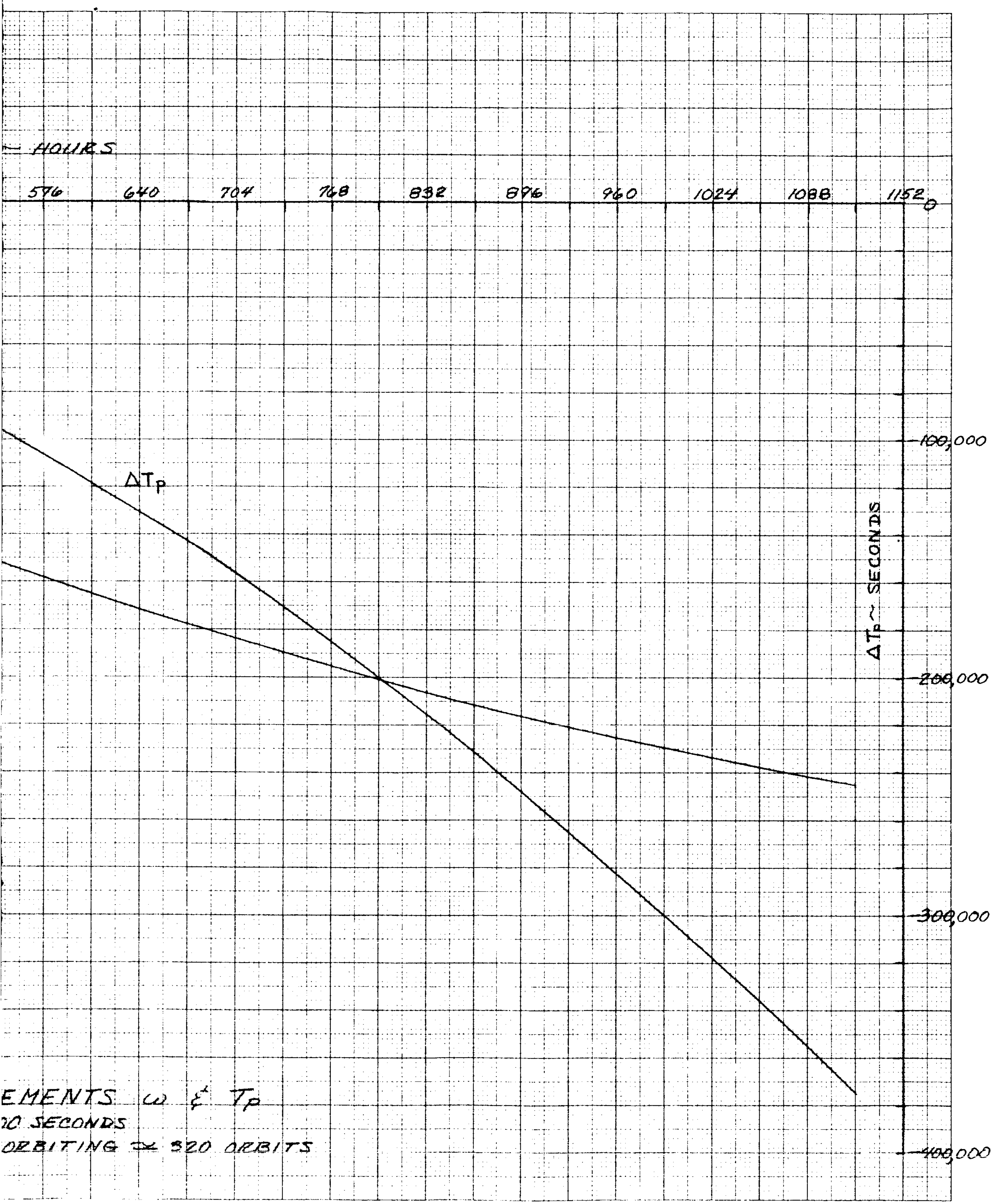

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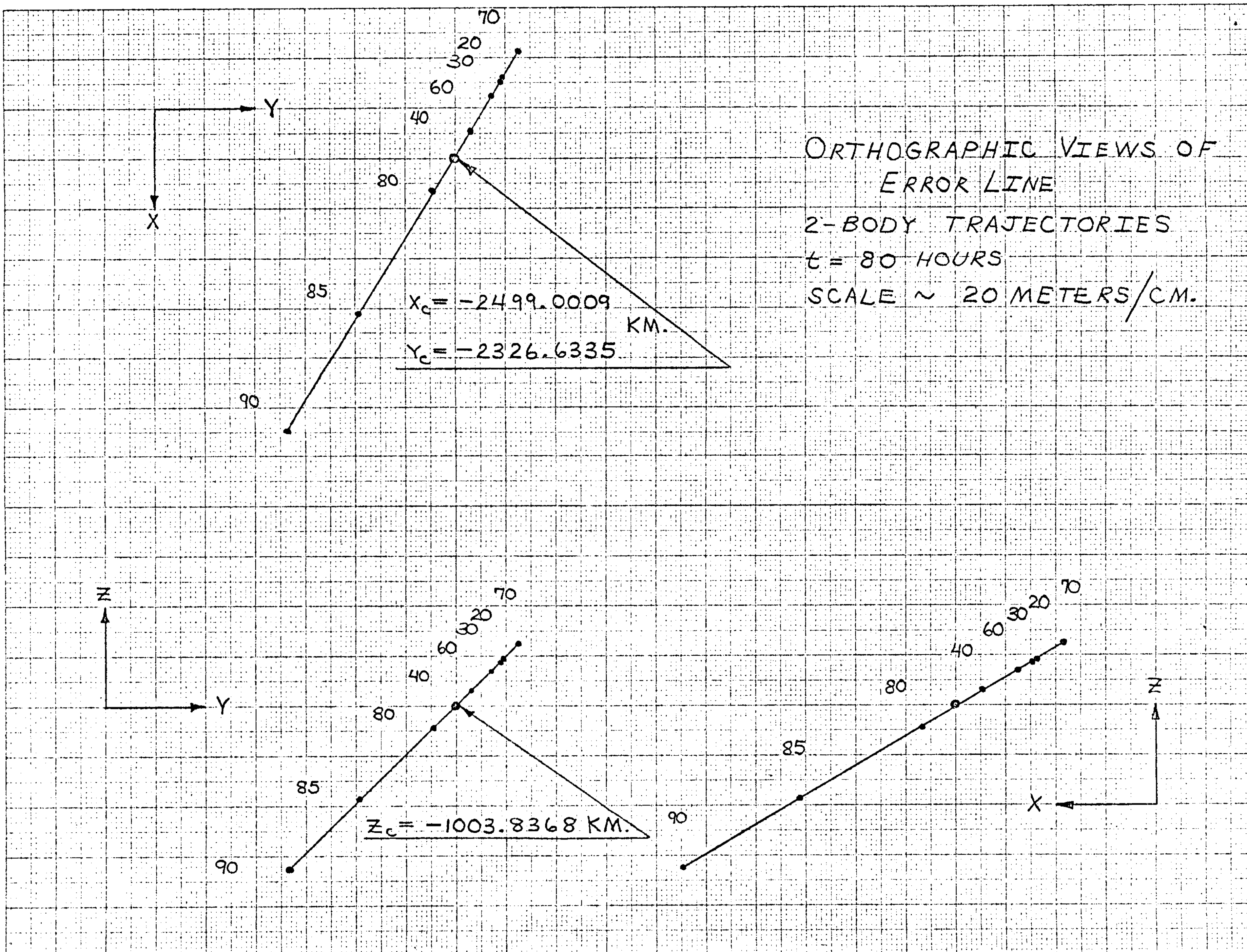
EMENTS ω & Tp
20 SECONDS
ORBITING ≈ 320 ORBITS

Fig. 4-2


ORTHOGRAPHIC VIEWS OF
ERROR LINE
2-BODY TRAJECTORIES
t = 80 HOURS
SCALE ~ 20 METERS/CM.
Xc = -2499.0009
KM.
Yc = -2326.6335
Zc = -1003.8368 KM.
X
Y
Z
70
20
30
60
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85
90

Fig. 5

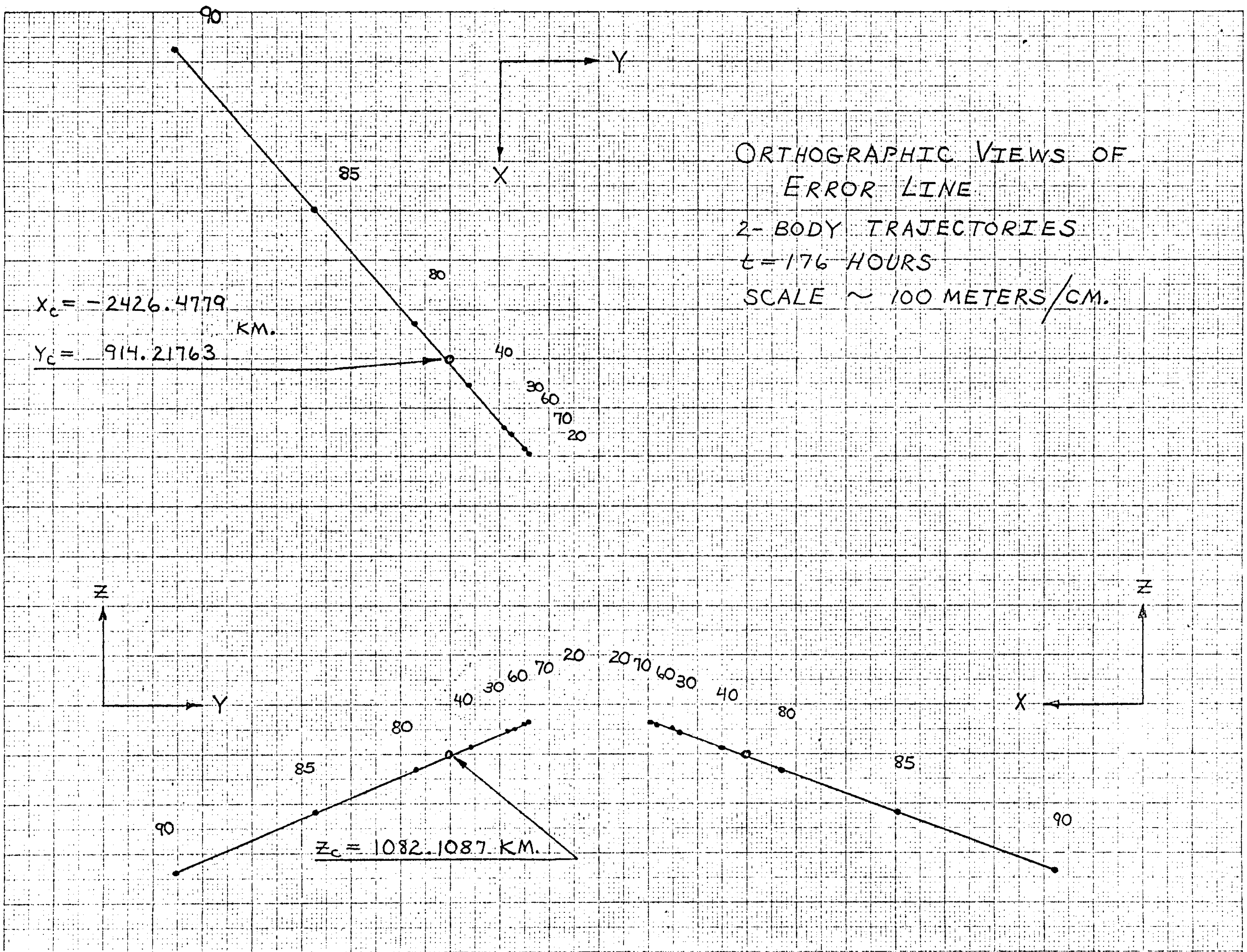
ORTHOGRAPHIC VIEWS OF
ERROR LINE
2- BODY TRAJECTORIES
t = 176 HOURS
SCALE ~ 100 METERS/CM.
X_c = −2426.4779
KM.
Y_c = 914.21763
Z_c = 1082.1087 KM.
X
Y
Z
90
85
80
40
30
60
70
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Fig. 6

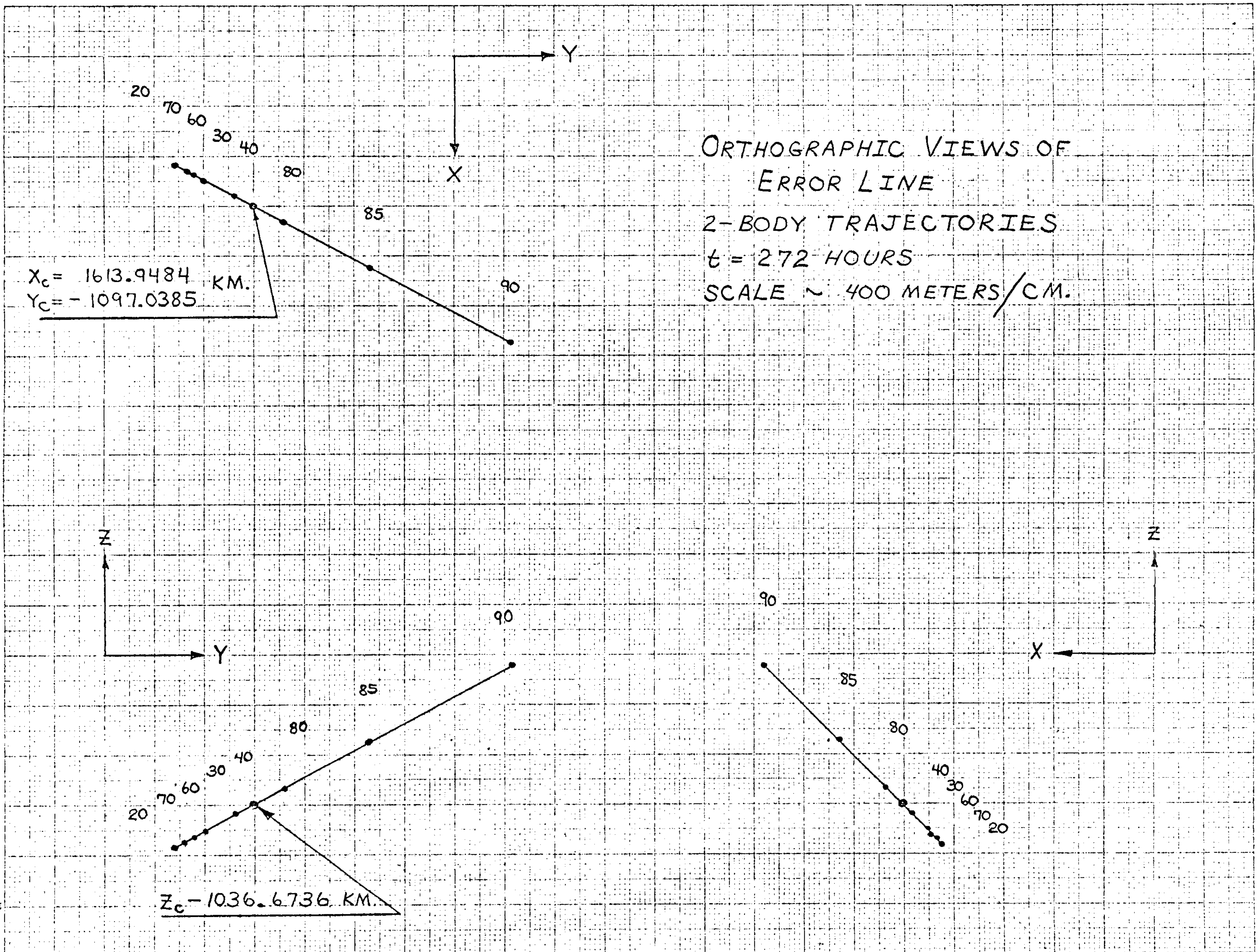
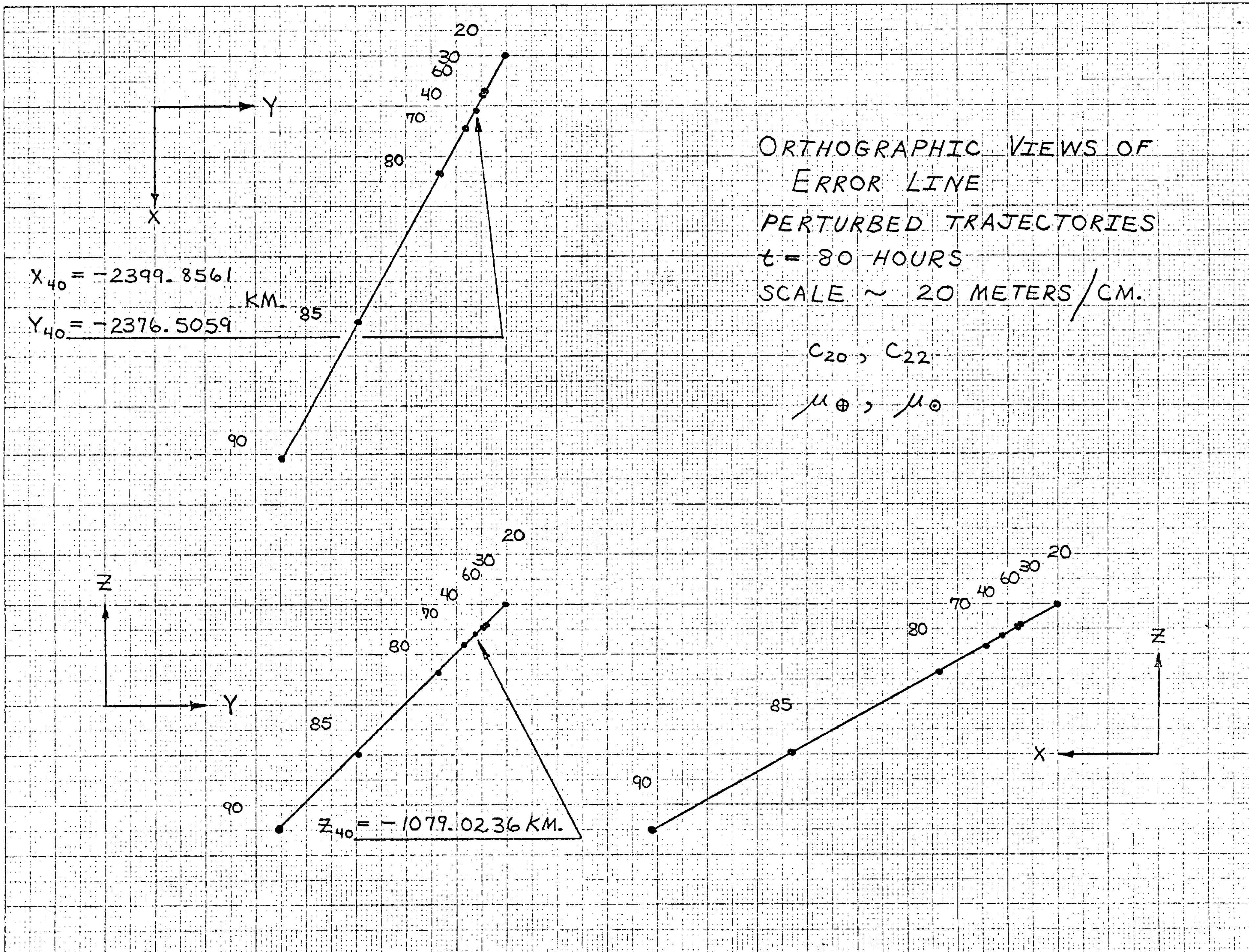

ORTHOGRAPHIC VIEWS OF
ERROR LINE
2-BODY TRAJECTORIES
t = 272 HOURS
SCALE ~ 400 METERS/CM.
X
Y
Z
20
70
60
30
40
80
85
90
Xc = 1613.9484 KM.
Yc = -1097.0385
Zc - 1036.6736 KM.

Fig. 7


ORTHOGRAPHIC VIEWS OF
ERROR LINE
PERTURBED TRAJECTORIES
t = 80 HOURS
SCALE ~ 20 METERS / CM.
C20, C22
μ⊕, μ☉
X40 = −2399.8561
Y40 = −2376.5059
KM.
Z40 = −1079.0236 KM.
X
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Z
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85
90

Fig. 8

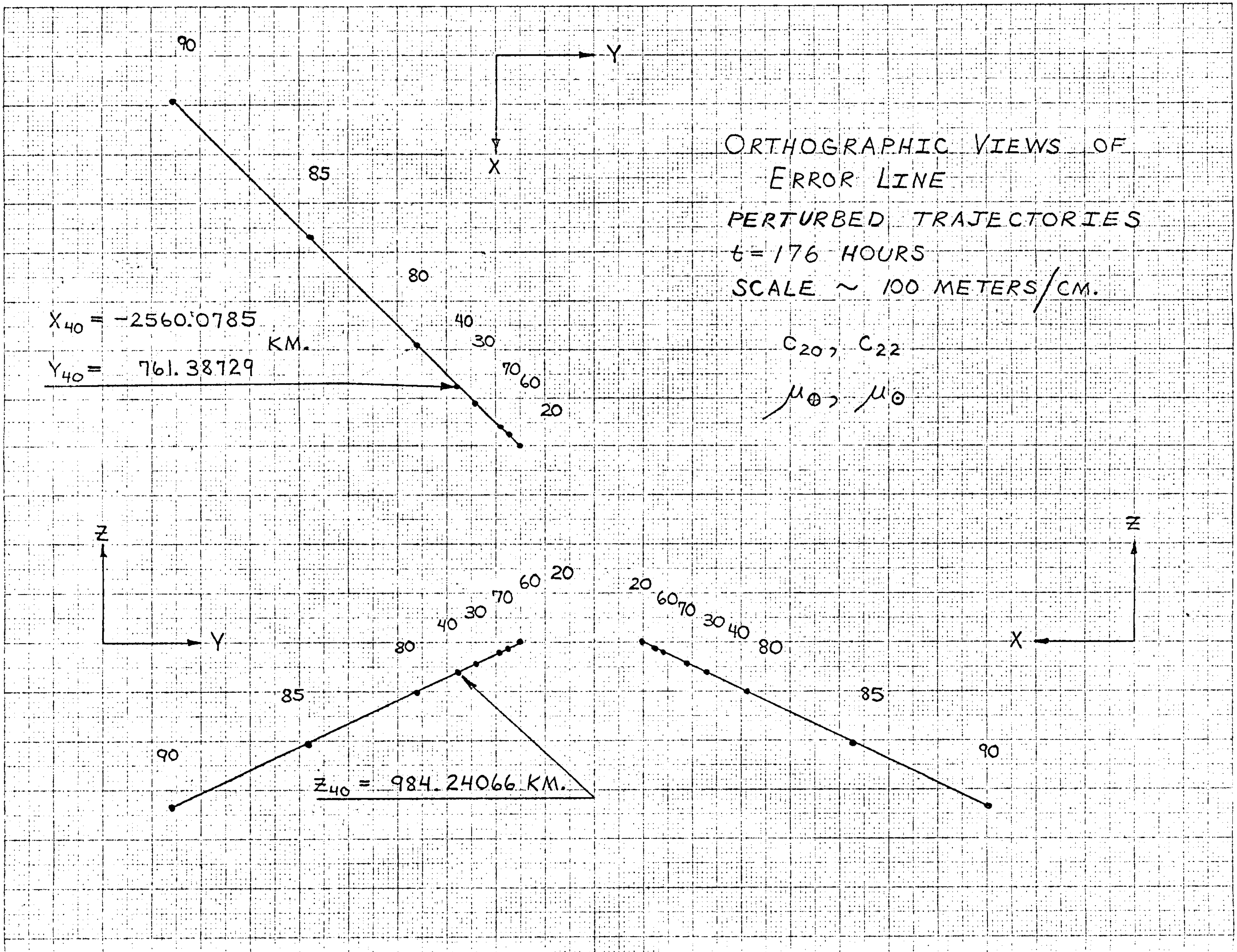

ORTHOGRAPHIC VIEWS OF
ERROR LINE
PERTURBED TRAJECTORIES
t = 176 HOURS
SCALE ~ 100 METERS/CM.
C_{20}, C_{22}
$\mu_{\oplus}$, $\mu_{\odot}$
$X_{40} = -2560.0785$ KM.
$Y_{40} = 761.38729$
$Z_{40} = 984.24066$ KM.
X
Y
Z
90
85
80
40
30
70
60
20

Fig. 9

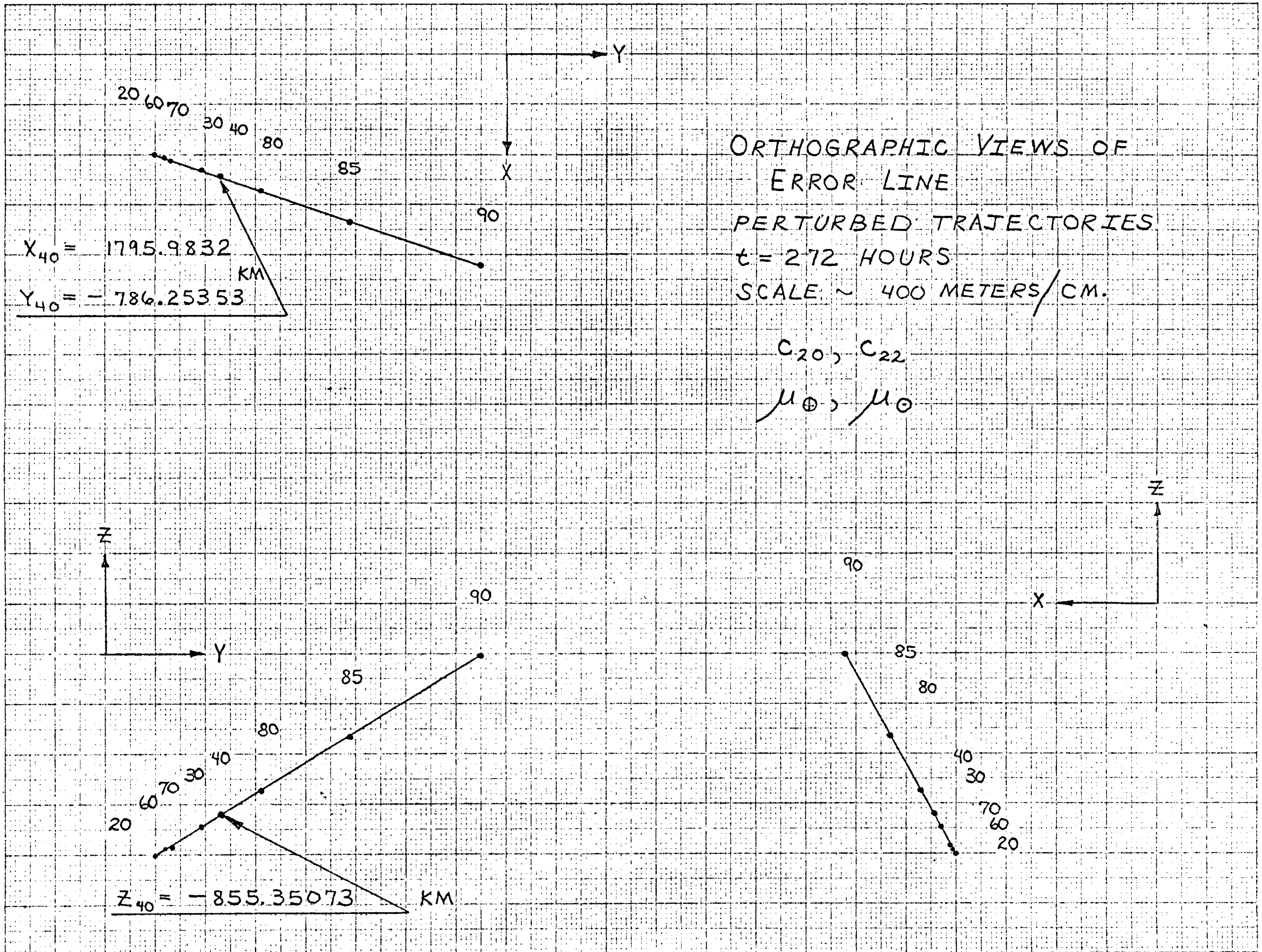

ORTHOGRAPHIC VIEWS OF
ERROR LINE
PERTURBED TRAJECTORIES
t = 272 HOURS
SCALE ~ 400 METERS/CM.
C_{20}, C_{22}
$\mu_{\oplus}$, $\mu_{\odot}$
X_{40} = 1795.9832
Y_{40} = −786.25353
KM
Z_{40} = −855.35073
KM
X
Y
Z
20 60 70 30 40 80 85 90

Fig. 10

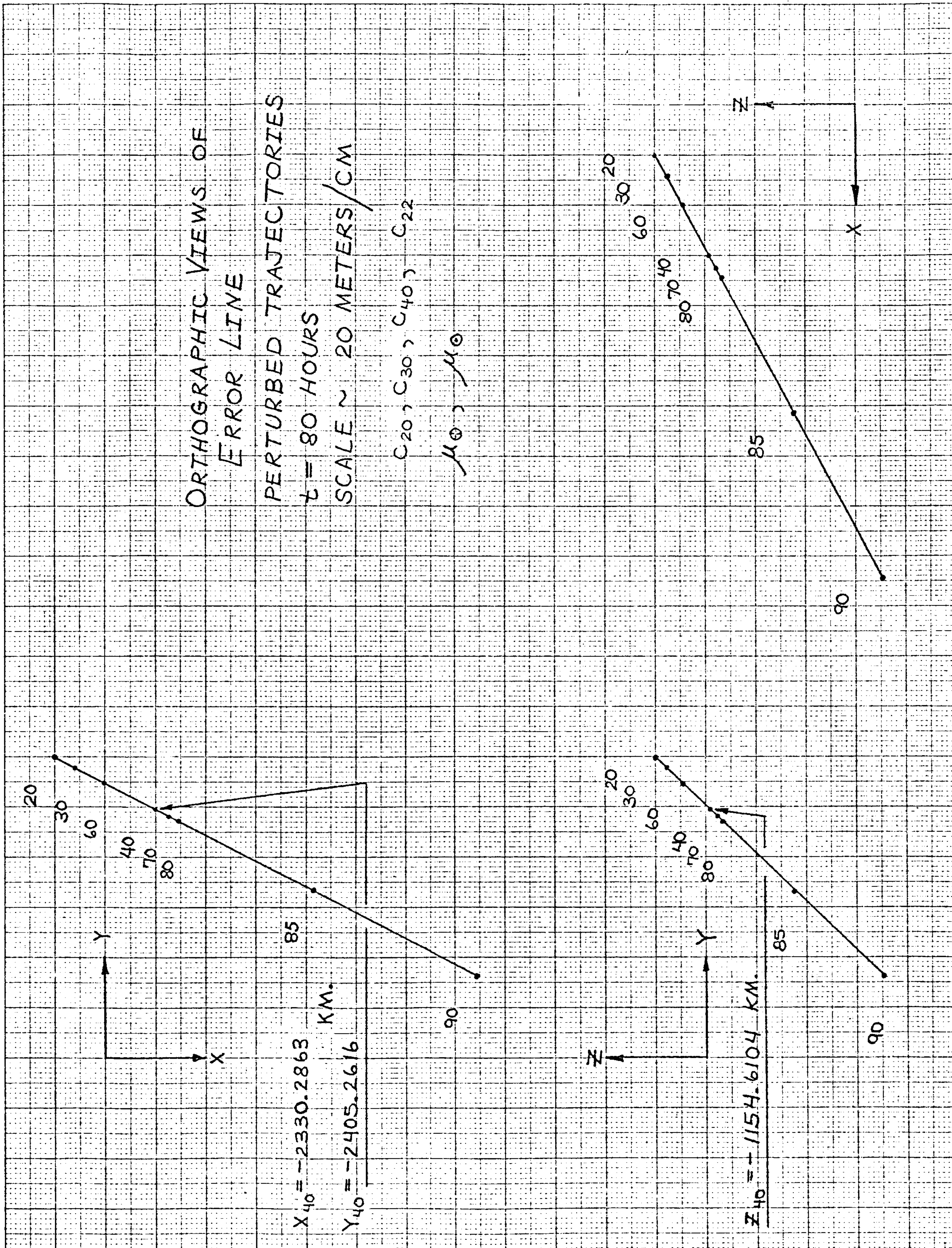

ORTHOGRAPHIC VIEWS OF
ERROR LINE
PERTURBED TRAJECTORIES
t = 80 HOURS
SCALE ~ 20 METERS/CM
C20, C30, C40, C22
μ⊕, μ☉
X40 = -2330.2863 KM.
Y40 = -2405.2616
Z40 = -1154.6104 KM.
X
Y
Z
20
30
60
40
70
80
85
90

Fig. 11

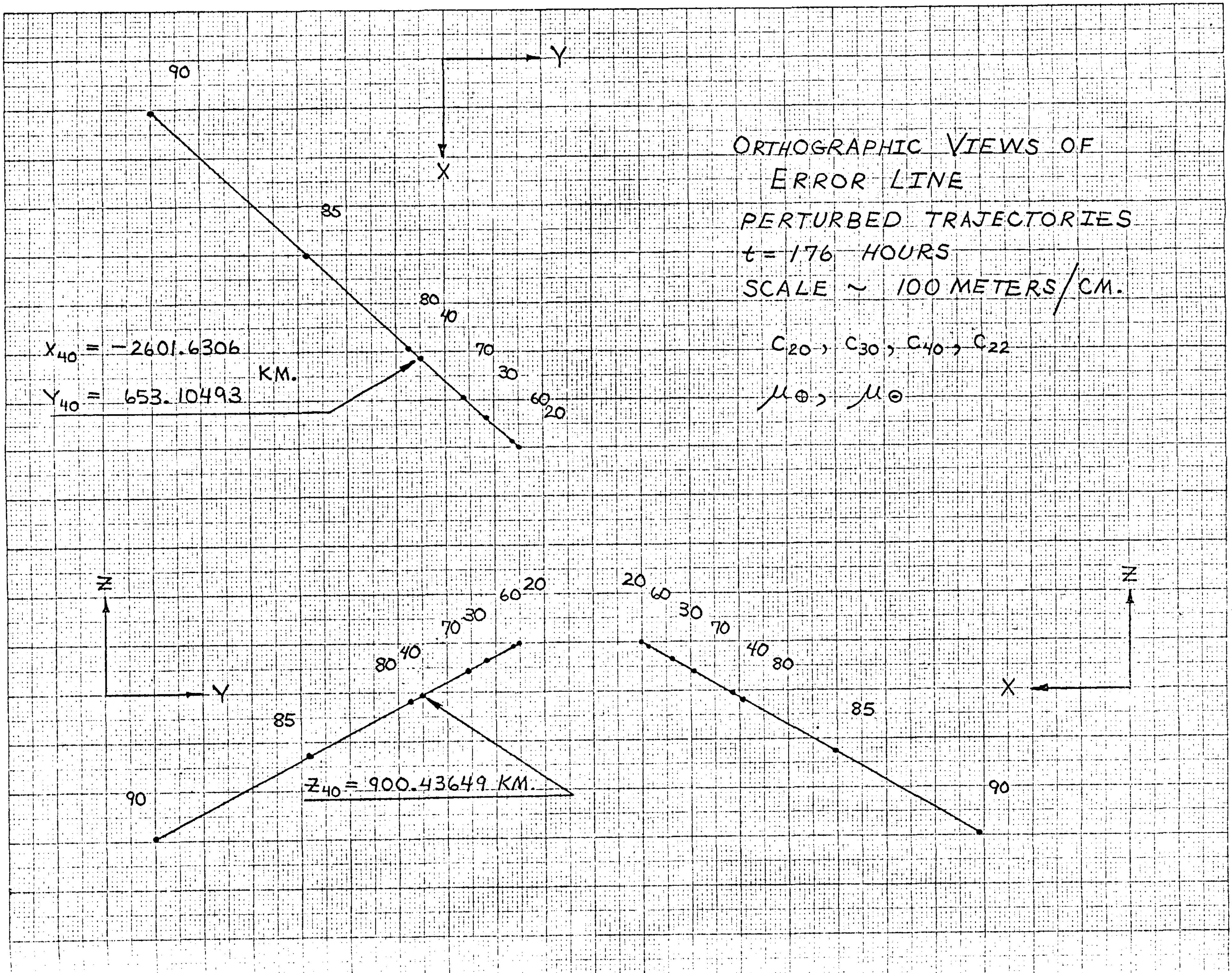

ORTHOGRAPHIC VIEWS OF
ERROR LINE
PERTURBED TRAJECTORIES
t = 176 HOURS
SCALE ~ 100 METERS/CM.
C_{20}, C_{30}, C_{40}, C_{22}
$\mu_\oplus$, $\mu_\odot$
$X_{40} = -2601.6306$ KM.
$Y_{40} = 653.10493$
$Z_{40} = 900.43649$ KM.
X
Y
Z

Fig. 12

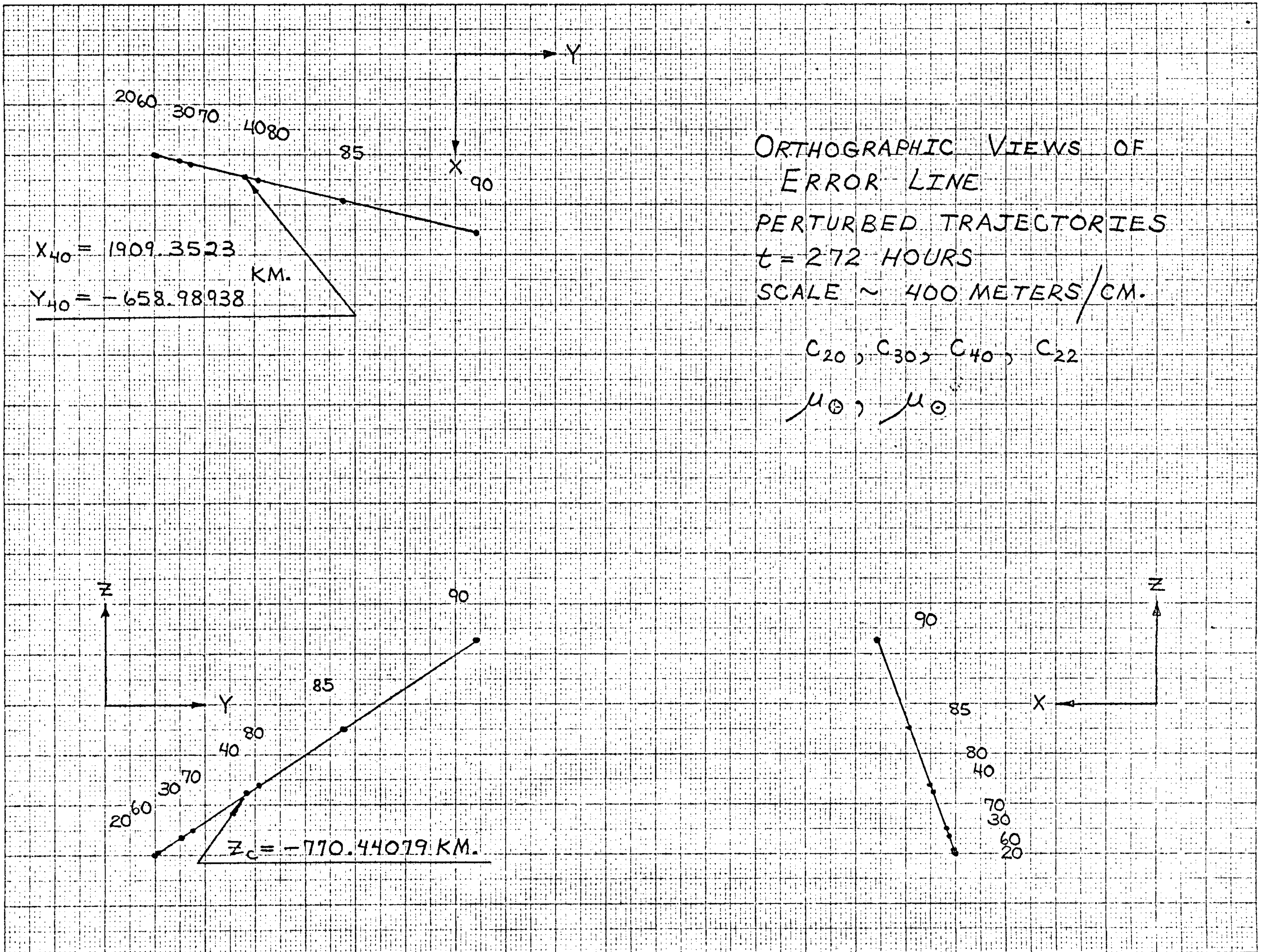
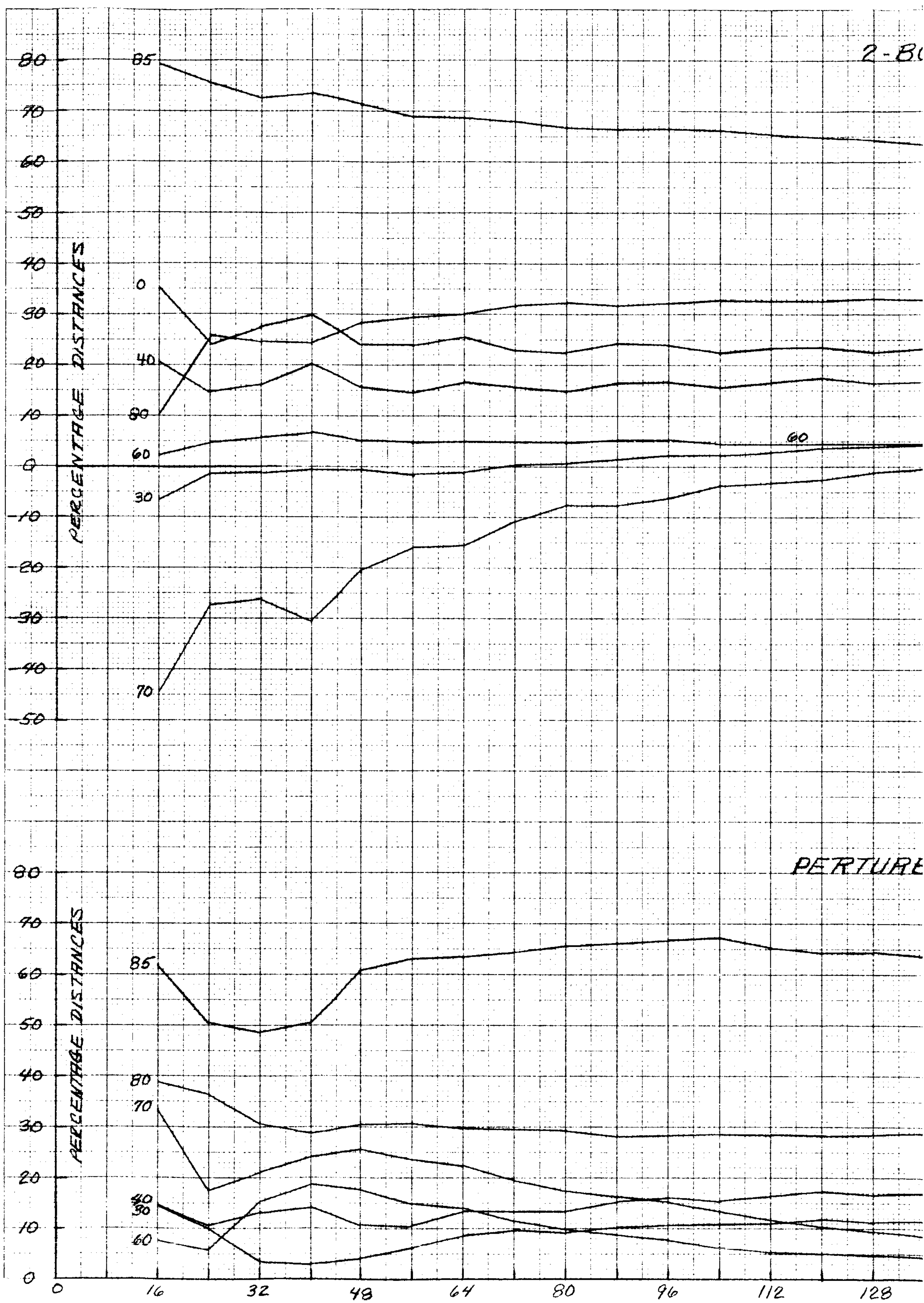

ORTHOGRAPHIC VIEWS OF
ERROR LINE
PERTURBED TRAJECTORIES
t = 272 HOURS
SCALE ~ 400 METERS/CM.
C20, C30, C40, C22
μ⊕, μ⊙
X40 = 1909.3523
Y40 = −658.98938
KM.
Zc = −770.44079 KM.
X
Y
Z
20 60
30 70
40 80
85
90

Fig. 13


2-B
PERCENTAGE DISTANCES
85
0
40
80
60
30
70
60
PERTURB
PERCENTAGE DISTANCES
85
80
70
40
30
60
0
16
32
48
64
80
96
112
128

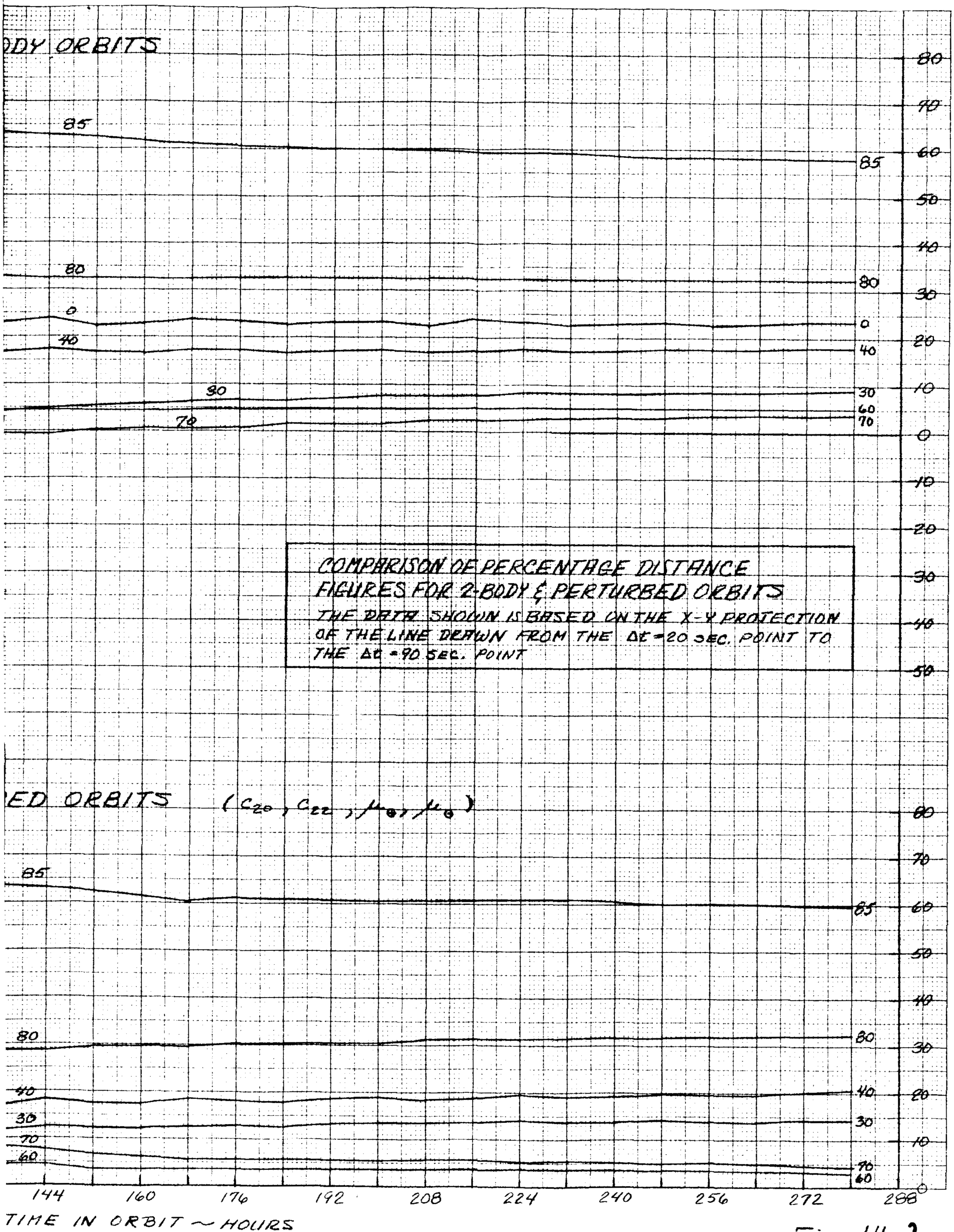

ODY ORBITS
COMPARISON OF PERCENTAGE DISTANCE
FIGURES FOR 2-BODY & PERTURBED ORBITS
THE DATA SHOWN IS BASED ON THE X-Y PROJECTION
OF THE LINE DRAWN FROM THE Δt = 20 SEC. POINT TO
THE Δt = 90 SEC. POINT
ED ORBITS (C_{20}, C_{22}, $\mu_{\oplus}$, $\mu_{\odot}$)
144
160
176
192
208
224
240
256
272
288
TIME IN ORBIT ~ HOURS

Fig. 14-2

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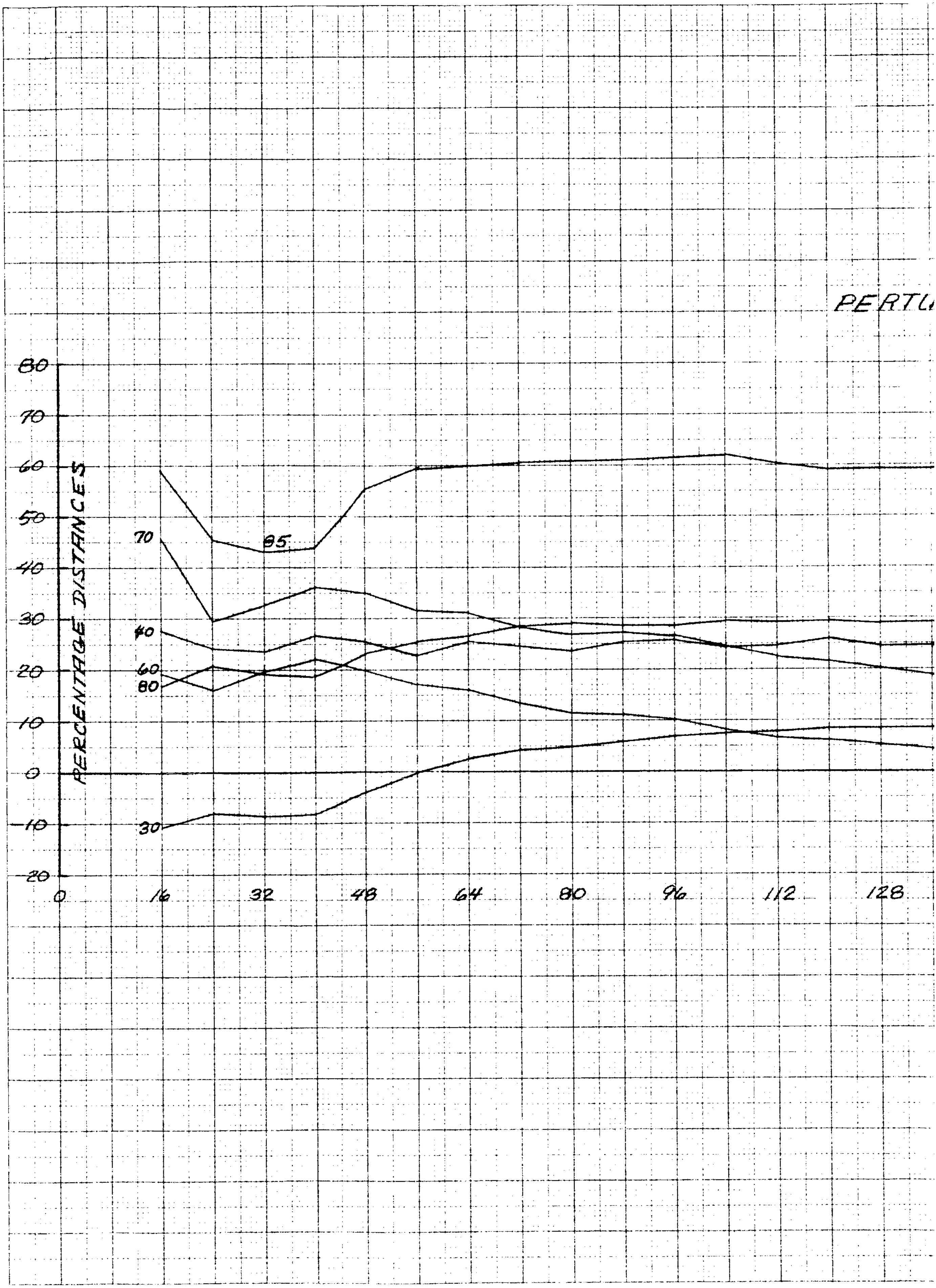
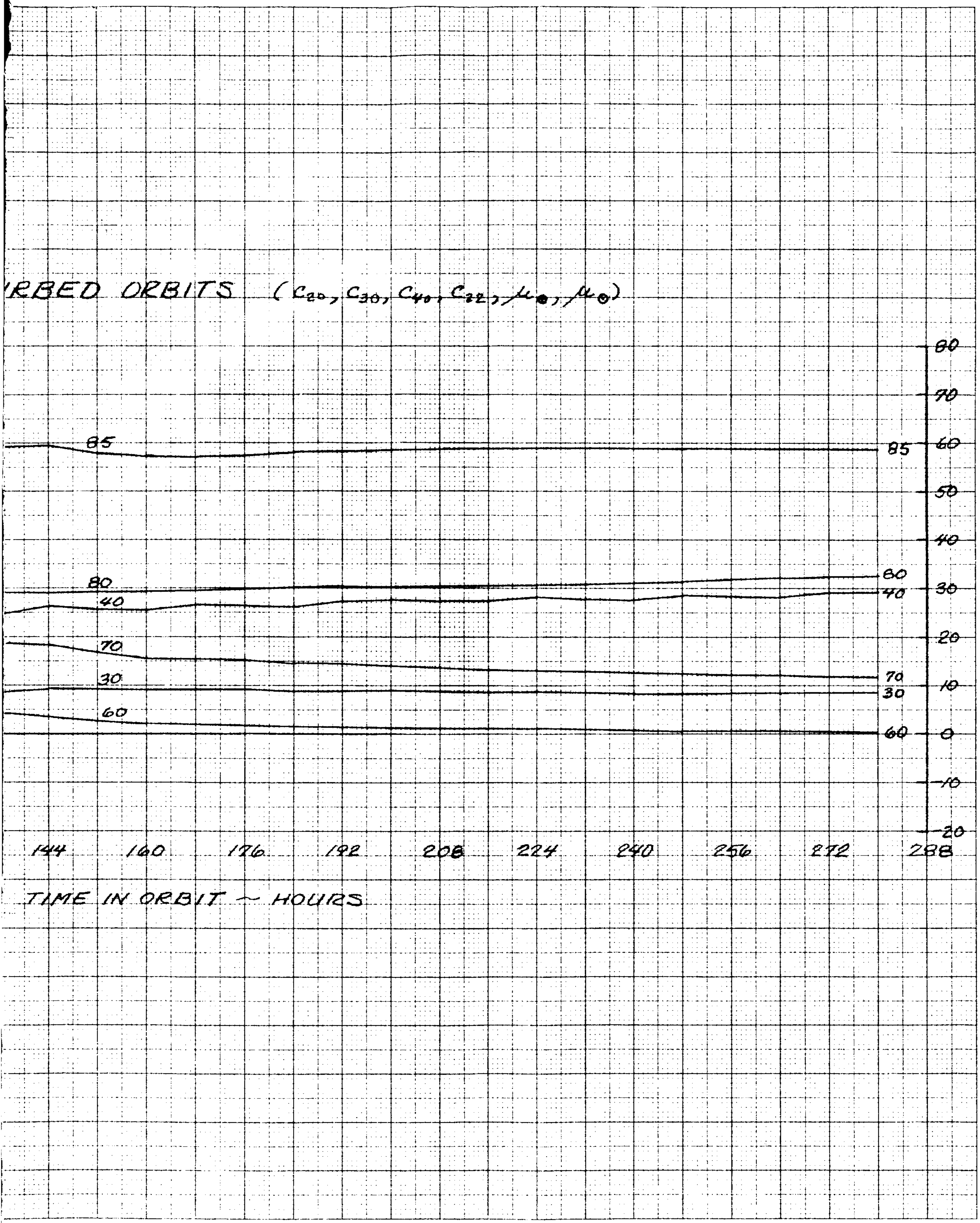
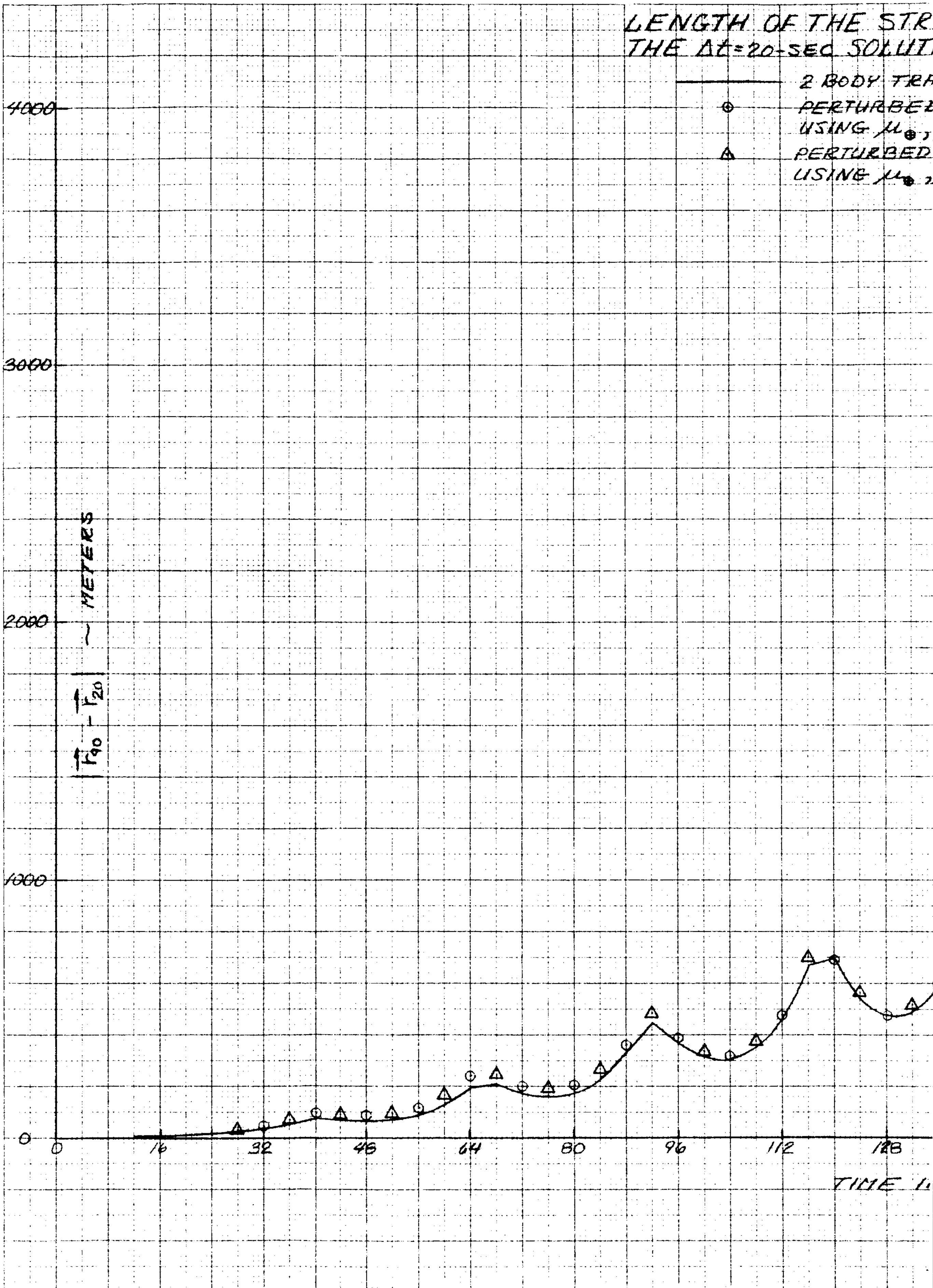
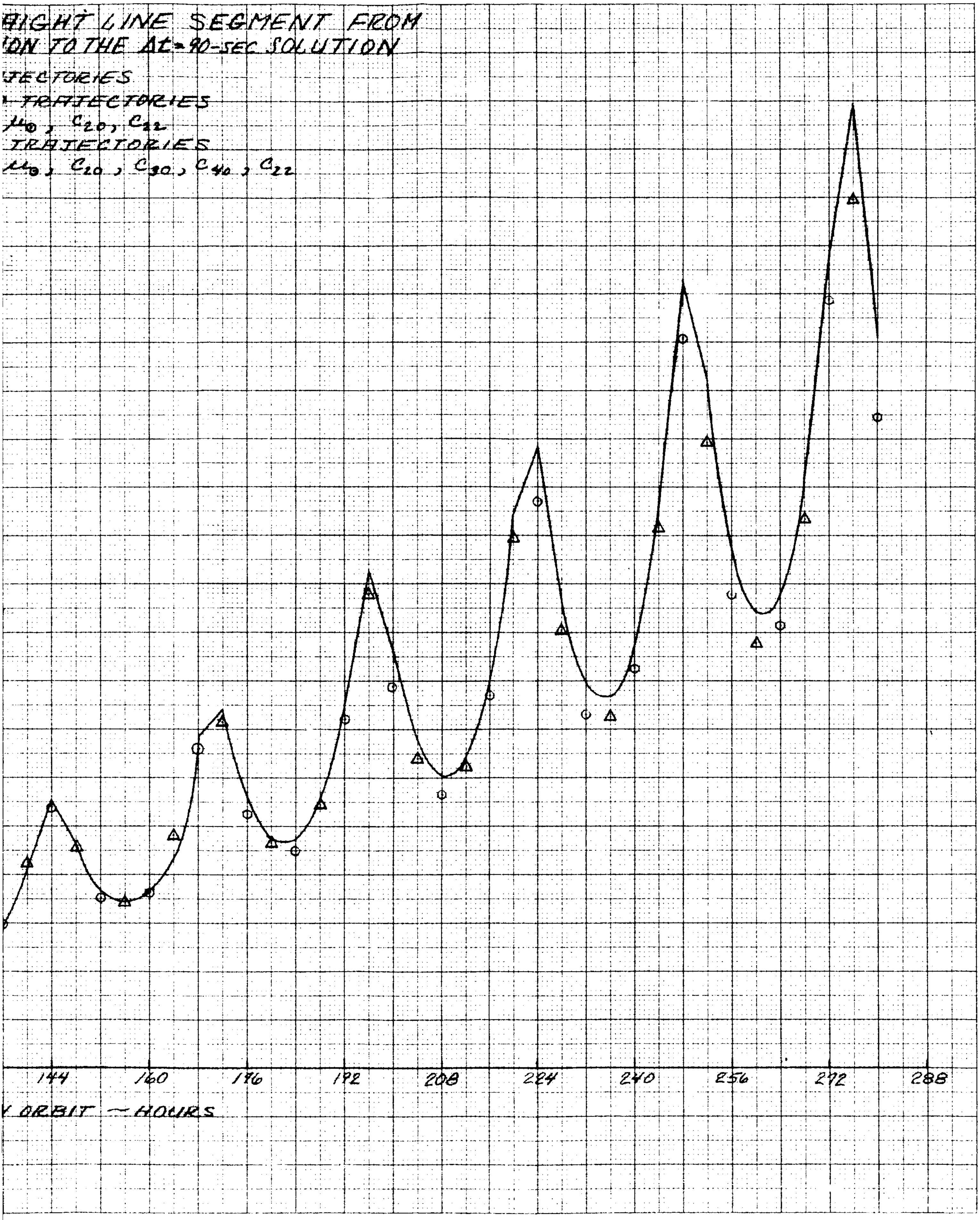




RBED ORBITS (C_{20}, C_{30}, C_{40}, C_{22}, μ, μ)
85
80
40
70
30
60
80
70
60
50
40
30
20
10
0
-10
-20
144
160
176
192
208
224
240
256
272
288
TIME IN ORBIT ~ HOURS

Fig. 15-2


LENGTH OF THE STR
THE Δt=20-SEC SOLUT
2 BODY TRA
PERTURBED
USING μ⊕,
PERTURBED
USING μ⊕
4000
3000
2000
1000
0
|r90 - r20| ~ METERS
0
16
32
48
64
80
96
112
128
TIME



RIGHT LINE SEGMENT FROM
ION TO THE Δt = 90-SEC SOLUTION
JECTORIES
TRAJECTORIES
$\mu_{\oplus}$, C_{20}, C_{22}
TRAJECTORIES
$\mu_{\oplus}$, C_{20}, C_{30}, C_{40}, C_{22}
144
160
176
192
208
224
240
256
272
288
ORBIT ~ HOURS

Fig. 16-2

7' Les 7

JET PROPULSION LABORATORY

TECHNICAL MEMORANDUM
312-620
Addendum No. 1
February 17, 1966

TO: Distribution

FROM: J. F. Gallagher

SUBJECT: A Few Further Results on Unperturbed Orbits and Accuracy Results on Perturbed Orbits for a Lunar Orbiter. Addendum No. 1 to TM 312-620, Accuracy and Running Time Study of SPACE for a Lunar Orbiter

DISTRIBUTION: J. Anderson, J. Brenkle (12), R. Broucke, C. Devine, W. Garrison, G. Gianopulos, T. Hamilton, R. Harris, C. Lawson, F. Lesh, J. Lorell, G. Null, C. Pfeiffer, A. Rosenberg, J. Scott, W. Sjogren, D. Trask, M. Warner, R. White, Trajectories and Performance Group

REFERENCES: 1. J. F. Gallagher, Accuracy and Running Time Study of SPACE for a Lunar Orbiter, TM 312-620, November 22, 1965

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